100/100 MARKS

ARTHAM
RESOURCE MATERIAL
NO. 1 EDUCATIONAL RESOURCES

NCERT

CHAPTER WISE TOPIC WISE

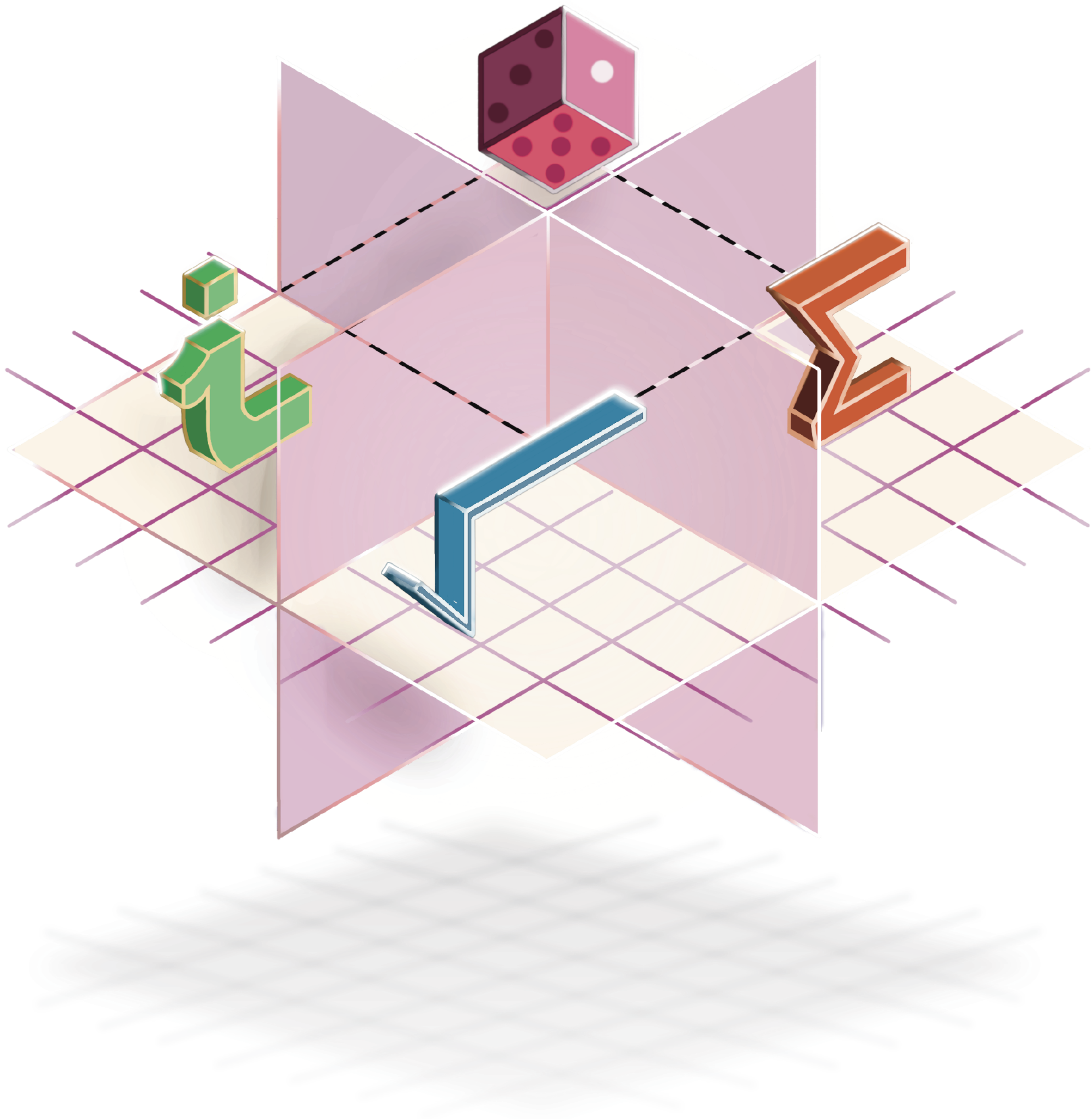
LINE BY LINE QUESTIONS

2024

BY SCHOOL OF EDUCATORS

Mathematics

Class 11



02

COMPLEX NUMBERS

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1. INTRODUCTION TO COMPLEX NUMBERS

A number of the form a + ib, where a, b ∈ R and $i = \sqrt{-1}$, is called a complex number and is denoted by 'z'.

$$\boxed{z = \boxed{a} + i\boxed{b}}$$
$$\downarrow \quad \downarrow$$
$$\mathrm{Re}(z) \quad \mathrm{Im}(z)$$

(i) If a = 0, then z is called a purely imaginary number.

(ii) If b = 0, then z is called a purely real number.

(iii) If b ≠ 0, then z is called an imaginary number.

NOTES :

1. Integral Powers of iota (i)

$$i^{4k+r} = \begin{cases} 1; & r = 0 \\ i; & r = 1 \\ -1; & r = 2 \\ -i; & r = 3 \end{cases}$$

2. $\sqrt{a}\,\sqrt{b} = \sqrt{ab}$ only if atleast one of either a or b is non-negative.

3. Real Numbers are a subset of complex numbers. $(R \subset C)$

2. ALGEBRA OF COMPLEX NUMBERS

2.1 Equality of complex number

$a + ib = c + id$

$\Leftrightarrow a = c \ \&\ b = d$

2.2 Let $z_1 = a + ib$ and $z_2 = c + id$ be two complex numbers where a, b, c, d ∈ R and $i = \sqrt{-1}$.

(a) Addition :

$$z_1 + z_2 = (a + ib) + (c + id) = (a + c) + (b + d)\,i$$

(b) Subtraction :

$$z_1 - z_2 = (a + ib) - (c + id) = (a - c) + (b - d)\,i$$

(c) Multiplication :

$$\begin{aligned} z_1 \cdot z_2 &= (a + ib)(c + id) \\ &= a(c + id) + ib(c + id) \\ &= ac + adi + bci + bdi^2 \\ &= ac - bd + (ad + bc)\,i \end{aligned}$$

$(\because i^2 = -1)$

(d) Division :

$$\frac{z_1}{z_2} = \frac{a+ib}{c+id} = \frac{a+ib}{c+id}\cdot\frac{c-id}{c-id}$$
$$= \left(\frac{ac+bd}{c^2+d^2}\right) + \left(\frac{bc-ad}{c^2+d^2}\right)i$$

3. CONJUGATE, MODULUS AND ARGUMENT OF A COMPLEX NUMBER

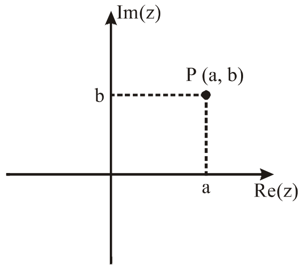
3.1 Conjugate of a Complex Number

For a given complex number z = a + ib,

its conjugate '$\bar{z}$' is defined as $\bar{z} = a - ib$

3.2 Argand Plane

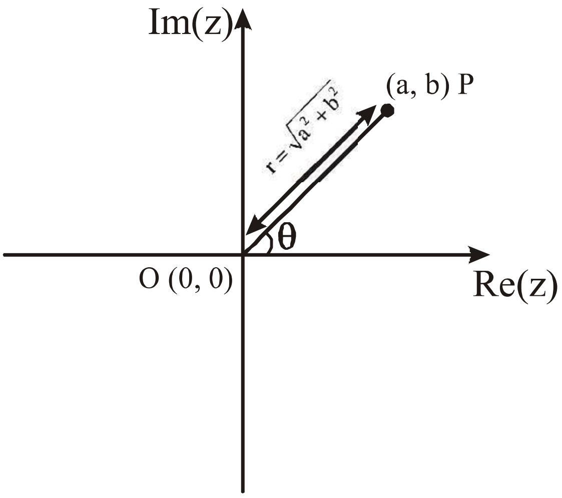
A complex number z = a + ib can be represented by a unique point P (a, b) in the Argand plane.



z = a + ib is represented by a point P (a, b)

3.3 Modulus and Argument of a Complex Number

If $z = a + ib$ is a complex number



(i) Distance of z from origin is called modulus of complex number z.

It is denoted by $r = |z| = \sqrt{a^2 + b^2}$

(ii) Here, θ i.e. angle made by OP with positive direction of real axis is called **argument of z**. It is denoted by arg(z) or amp (z).

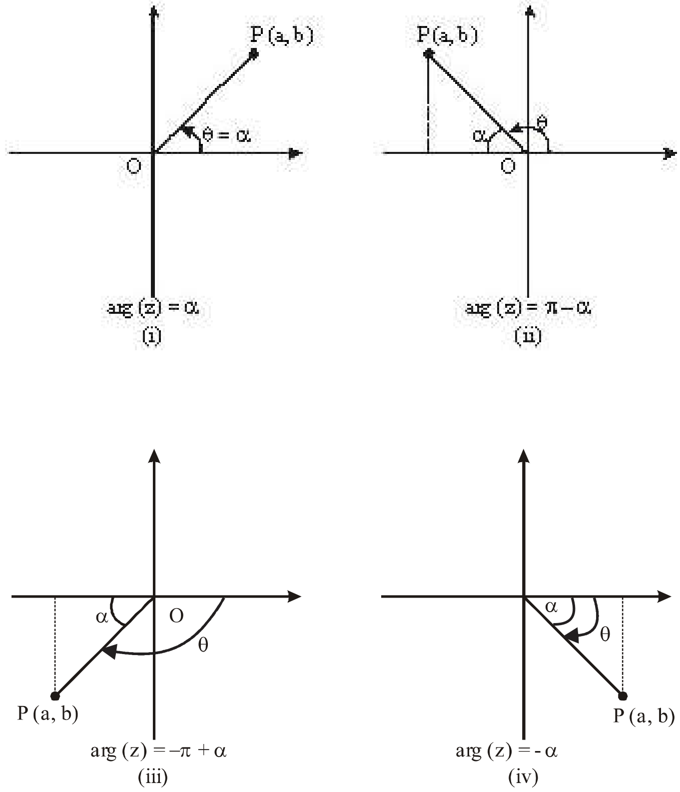
NOTES :

1. $z_1 > z_2$ or $z_1 < z_2$ has no meaning but $|z_1| > |z_2|$ or $|z_1| < |z_2|$ holds meaning.
2. $|z_1 - z_2|$ represents distance between z_1 and z_2 on Argand Plane.

3.4 Principal Argument

The argument 'θ' of complex number z = a + ib is called principal argument of z if $-\pi < \theta \le \pi$.

Let $\tan \alpha = \left|\frac{b}{a}\right|$, and θ be the principal argument of z.



NOTES :

Argument is not defined for 0.

4. PROPERTIES OF MODULUS, ARGUMENT AND CONJUGATE

1. $\overline{(\overline{z})} = z$
2. $z + \overline{z} = 2\,\text{Re}(z) \Rightarrow z + \overline{z} = 0,$ if z is purely imaginary
3. $z - \overline{z} = 2i\,\text{Im}(z) \Rightarrow z = \overline{z},$ if z is purely real
4. $\overline{z_1 \pm z_2} = \overline{z}_1 \pm \overline{z}_2$
5. $\overline{z_1 z_2} = \overline{z}_1\,\overline{z}_2$ *and* $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z}_1}{\overline{z}_2}\ (z_2 \ne 0)$
6. $|z| = 0 \Rightarrow z = 0$
7. $z\overline{z} = |z|^2$

8. $|z_1 z_2| = |z_1||z_2|$; $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$

9. $|\bar{z}| = |z| = |-z|$

10. $|z_1 \pm z_2|^2 = |z_1|^2 + |z_2|^2 \pm 2\,\mathrm{Re}\,(z_1\,\bar{z}_2)$

$= |z_1|^2 + |z_2|^2 \pm 2|z_1||z_2|\cos(\theta_1 - \theta_2)$

11. $||z_1| - |z_2|| \le |z_1 + z_2| \le |z_1| + |z_2|$ (Triangle Inequality)

12. $||z_1| - |z_2|| \le |z_1 - z_2| \le |z_1| + |z_2|$ (Triangle Inequality)

13. $|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = (a^2 + b^2)(|z_1|^2 + |z_2|^2)$

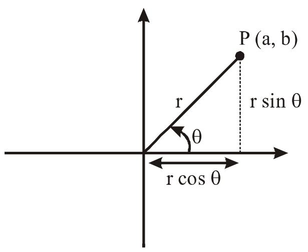
14. $\mathrm{amp}\,(z_1 . z_2) = \mathrm{amp}\ z_1 + \mathrm{amp}\,z_2 + 2k\pi$; $k \in I$

15. $\mathrm{amp}\left(\dfrac{z_1}{z_2}\right) = \mathrm{amp}\,z_1 - \mathrm{amp}\,z_2 + 2k\pi$; $k \in I$

16. $\mathrm{amp}(z^n) = n\,\mathrm{amp}(z) + 2k\pi$; $k \in I$

17. $\mathrm{amp}(\bar{z}) = -\mathrm{amp}(z) + 2k\pi,\ k \in I$

5. POLAR/TRIGONOMETRIC FORM OF A COMPLEX NUMBER



$a = r\cos\theta$ & $b = r\sin\theta$;

where $r = |z|$ and $\theta = \arg(z)$

$\therefore\ z = a + ib$

$= r(\cos\theta + i\sin\theta) = r\,\mathrm{cis}\,\theta$

6. EULER'S FORM OF A COMPLEX NUMBER

$z = re^{i\theta}$ is known as Euler's form; where

$r = |z|$ & $\theta = \arg(z)$

7. VECTORIAL REPRESENTATION OF A COMPLEX NUMBER

Every complex number can be considered as if it is the position vector of that point. If the point P represents the complex number z then,

$\overrightarrow{OP} = z$ & $|\overrightarrow{OP}| = |z|$.

8. DE-MOIVRE'S THEOREM

Case 1

(i) If n is an integer, then

$(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$

(ii) $(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)..........(\cos\theta_n + i\sin\theta_n)$

$= \cos(\theta_1 + \theta_2 + ... + \theta_n) + i\sin(\theta_1 + \theta_2 + ... + \theta_n)$

Case 2

If n is a rational number (but not an integer),

n can be written as p/q, where p,q, $\in$ I and $q \neq 0$

$$(\cos\theta + i\sin\theta)^{p/q} = \cos\left(\frac{2k\pi + p\theta}{q}\right) + i\sin\left(\frac{2k\pi + p\theta}{q}\right)$$

where $k = 0, 1, 2, 3, ..., (q - 1)$

9. CUBE ROOTS OF UNITY

Roots of the equation $x^3 = 1$ are called cube roots of unity.

$x^3 - 1 = 0$

$(x - 1)(x^2 + x + 1) = 0$

$x = 1$ or $x^2 + x + 1 = 0$

i.e $x = \underbrace{\dfrac{-1+\sqrt{3}i}{2}}_{\omega}$ or $x = \underbrace{\dfrac{-1-\sqrt{3}i}{2}}_{\omega^2}$

(i) The cube roots of unity are $1, \dfrac{-1+i\sqrt{3}}{2}, \dfrac{-1-i\sqrt{3}}{2}$.

(ii) $\omega^3 = 1,\ \omega^{3r} = 1,\ \omega^{3r+1} = \omega,\ \omega^{3r+2} = \omega^2\ (r \in I)$

(iii) If ω is one of the imaginary cube roots of unity then $1 + \omega + \omega^2 = 0$.

(iv) In general $1 + \omega^r + \omega^{2r} = 0$; where $r \in I$ but is not a multiple of 3.

(v) In polar form the cube roots of unity are :

$$\cos 0 + i \sin 0 \ ; \ \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}, \quad \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$$

(vi) The three cube roots of unity when plotted on the argand plane constitute the verties of an equilateral triangle.

(vii) The following factorisation should be remembered :

$x^2 + x + 1 = (x - \omega)(x - \omega^2)$;

$a^3 - b^3 = (a - b)(a - \omega b)(a - \omega^2 b)$;

$a^3 + b^3 = (a + b)(a + \omega b)(a + \omega^2 b)$;

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a + \omega b + \omega^2 c)(a + \omega^2 b + \omega c)$

10. IMPORTANT IDENTITIES

(i) $x^2 + x + 1 = (x - \omega)(x - \omega^2)$

(ii) $x^2 - x + 1 = (x + \omega)(x + \omega^2)$

(iii) $x^2 + xy + y^2 = (x - y\omega)(x - y\omega^2)$

(iv) $x^2 - xy + y^2 = (x + \omega y)(x + y\omega^2)$

(v) $x^2 + y^2 = (x + iy)(x - iy)$

(vi) $x^3 + y^3 = (x + y)(x + y\omega)(x + y\omega^2)$

(vii) $x^3 - y^3 = (x - y)(x - y\omega)(x - y\omega^2)$

(viii) $x^2 + y^2 + z^2 - xy - yz - zx = (x + y\omega + z\omega^2)(x + y\omega^2 + z\omega)$

or $(x\omega + y\omega^2 + z)(x\omega^2 + y\omega + z)$

or $(x\omega + y + z\omega^2)(x\omega^2 + y + z\omega)$.

(ix) $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x + \omega y + \omega^2 z)(x + \omega^2 y + \omega z)$

NOTES :

If z_1 is a root of a polynomial with real coefficients, then $\bar{z}_1$ is also one of its roots.

11. 'n' nth ROOTS OF UNITY

Solution of equation $x^n = 1$ is given by

$$x = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n} \quad ; k = 0, 1, 2, ..., (n-1)$$

$$= e^{i\left(\frac{2k\pi}{n}\right)} \quad ; k = 0, 1, ..., (n-1)$$

NOTES :

1. We may take any n consecutive integral values of k to get 'n' nth roots of unity.
2. Sum of 'n' nth roots of unity is zero, $n \in N$
3. The points represented by 'n' nth roots of unity are located at the vertices of regular polygon of n sides inscribed in a unit circle, centred at origin & one vertex being on +ve real axis.

Properties :

If $1, \alpha_1, \alpha_2, \alpha_3 ..... \alpha_{n-1}$ are the n, nth root of unity then:

(i) They are in G.P. with common ratio $e^{i(2\pi/n)}$

(ii) $1^p + \alpha_1^p + \alpha_2^p + ... + \alpha_{n-1}^p = \begin{cases} 0, & \text{if } p \neq kn \\ n, & \text{if } p = kn \end{cases}$ where $k \in I$

(iii) $(1 - \alpha_1)(1 - \alpha_2) ...... (1 - \alpha_{n-1}) = n$

(iv) $(1 + \alpha_1)(1 + \alpha_2) .... (1 + \alpha_{n-1}) = \begin{cases} 0, & \text{if } n \text{ is even} \\ 1, & \text{if } n \text{ is odd} \end{cases}$

(v) $1 . \alpha_1 . \alpha_2 . \alpha_3 .... \alpha_{n-1} = \begin{cases} -1, & \text{if } n \text{ is even} \\ 1, & \text{if } n \text{ is odd} \end{cases}$

NOTES :

(i) $\cos\theta + \cos 2\theta + \cos 3\theta + ... + \cos n\theta = \dfrac{\sin(n\theta/2)}{\sin(\theta/2)} \cos\left(\dfrac{n+1}{2}\right)\theta.$

(ii) $\sin\theta + \sin 2\theta + \sin 3\theta + ... + \sin n\theta = \dfrac{\sin(n\theta/2)}{\sin(\theta/2)} \sin\left(\dfrac{n+1}{2}\right)\theta.$

12. SQUARE ROOT OF A COMPLEX NUMBER

Let $x + iy = \sqrt{a + ib}$, Squaring both sides, we get

$(x + iy)^2 = a + ib$

i.e. $x^2 - y^2 = a, 2xy = b$

By using relation

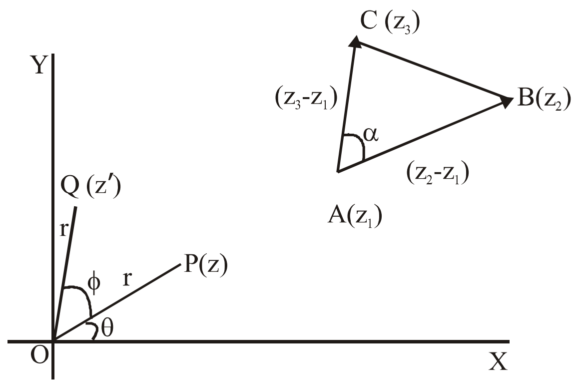
$(x^2 + y^2)^2 = (x^2 - y^2) + 4x^2y^2$

we can find x^2+y^2 and then solve x^2+y^2 and $x^2 - y^2$ to get values of x and y.

we obtain

$$\sqrt{a+ib} = \pm\left(\sqrt{\frac{\sqrt{a^2+b^2}+a}{2}} + i\frac{b}{|b|}\sqrt{\frac{\sqrt{a^2+b^2}-a}{2}}\right)$$

13. ROTATION THEOREM



1. If $\vec{OP} = z = r e^{i\theta}$ then $\vec{OQ} = z_1 = r e^{i(\theta+\phi)} = z \cdot e^{i\phi}$.

If $\vec{OP}$ and $\vec{OQ}$ are of equal magnitude then $\vec{OQ} = \vec{OP}\, e^{i\phi}$. Thus, to rotate a complex number z counter clockwise by ϕ without changing its magnitude, we multiply it with $e^{i\phi}$.

2. If z_1, z_2, z_3 are three vertices of a triangle ABC described in the counter-clockwise sense, then

$$\frac{z_3-z_1}{z_2-z_1} = \frac{AC}{AB}(\cos\alpha + i\sin\alpha) = \frac{AC}{AB}.e^{i\alpha} = \frac{|z_3-z_1|}{|z_2-z_1|}.e^{i\alpha}$$

(Rotating AB about A by angle α to get AC)

14. GEOMETRY OF COMPLEX NUMBERS

14.1 Locus from inspection based on modulus, argument and rotation

(i) $|z-z_o| = a$ represents circumference of circle, centred at z_o, radius a.

(ii) $|z-z_o| < a$ represents interior of circle

(iii) $|z-z_o| > a$ represents exterior of this circle.

(iv) $|z-z_1| = |z-z_2|$ represents $\perp$ bisector of segment with end points z_1 & z_2.

(v) $\left|\dfrac{z-z_1}{z-z_2}\right| = k$ represents : $\begin{Bmatrix} \text{circle, } k \neq 1, k > 0 \\ \perp \text{bisector, } k = 1 \end{Bmatrix}$

(vi) arg (z) = θ is a ray starting from origin (excluded) inclined at an ∠θ with positive real axis.

(vii) arg $(z - z_1)$ is a ray starting from z_1 (excluded) inclined at an ∠θ with positive real axis.

(viii) $|z - z_1| + |z - z_2| = k$ is

(a) an ellipse with foci z_1 and z_2 if $k > |z_1 - z_2|$.

(b) Line segment joining z_1 and z_2 if $k = |z_1 - z_2|$

(c) No point if $k < |z_1 - z_2|$

(ix) $||z - z_1| - |z - z_2|| = k$ is a hyperbola if $k < |z_1 - z_2|$

NOTES :

To convert complex equation to cartesian equation, we can replace z by x + iy.

14.2 Standard Results

(i) If z_1 and z_2 are two complex numbers, then the distance between z_1 and z_2 is $|z_2 - z_1|$.

(ii) Segment joining points A (z_1) and B(z_2) is divided by point P (z) in the ratio $m_1 : m_2$

then $z = \dfrac{m_1z_2 + m_2z_1}{m_1 + m_2}$, m_1 and m_2 are real.

For external division $z = \dfrac{m_1z_2 - m_2z_1}{m_1 - m_2}$

(iii) Centroid (z) of triangle with vertices z_1, z_2, z_3 is given by

$$z = \frac{z_1+z_2+z_3}{3}$$

14.3 General Equations

(i) The equation of the line joining z_1 and z_2 is given by

$$\begin{vmatrix} z & \bar{z} & 1 \\ z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \end{vmatrix} = 0 \text{ (non parametric form)}$$

Or

$z = z_1 + t(z_1 - z_2)$, where t is a real parameter

Or

$$\frac{z - z_1}{\bar{z} - \bar{z}_1} = \frac{z - z_2}{\bar{z} - \bar{z}_2}$$

(ii) $\bar{a}z + a\bar{z} + b = 0$ represents general form of line. $(b \in R, a \neq 0)$

(iii) The general equation of circle is :

$z\bar{z} + a\bar{z} + \bar{a}z + b = 0$ (where b is real number).

Centre : (–a) & radius $\sqrt{|a|^2 - b} = \sqrt{a\bar{a} - b}$.

(iv) Circle described on line segment joining z_1 & z_2 as diameter is :

$$(z - z_1)(\bar{z} - \bar{z}_2) + (z - z_2)(\bar{z} - \bar{z}_1) = 0.$$

(v) Four pts. z_1, z_2, z_3, z_4 in anticlockwise order will be concylic, if & only if

$$\theta = \arg\left(\frac{z_2 - z_4}{z_1 - z_4}\right) = \arg\left(\frac{z_2 - z_3}{z_1 - z_3}\right)$$

$$\Rightarrow \arg\left(\frac{z_2 - z_4}{z_1 - z_4}\right) - \arg\left(\frac{z_2 - z_3}{z_1 - z_3}\right) = 2n\pi \ ; (n \in I)$$

$$\Rightarrow \arg\left[\left(\frac{z_2 - z_4}{z_1 - z_4}\right)\left(\frac{z_1 - z_3}{z_2 - z_3}\right)\right] = 2n\pi$$

$$\Rightarrow \left(\frac{z_2 - z_4}{z_1 - z_4}\right) \times \left(\frac{z_1 - z_3}{z_2 - z_3}\right) \text{ is real \& positive.}$$

(vi) If z_1, z_2, z_3 are the vertices of an equilateral triangle where z_0 is its circumcentre then

(a) $\dfrac{1}{z_2 - z_3} + \dfrac{1}{z_3 - z_1} + \dfrac{1}{z_1 - z_2} = 0$

(b) $z_1^2 + z_2^2 + z_3^2 - z_1 z_2 - z_2 z_3 - z_3 z_1 = 0$

(c) $z_1^2 + z_2^2 + z_3^2 = 3z_0^2$

(vii) If A, B, C & D are four points representing the complex numbers z_1, z_2, z_3 & z_4 then

AB || CD if $\dfrac{z_4 - z_3}{z_2 - z_1}$ is purely real ;

AB $\perp$ CD if $\dfrac{z_4 - z_3}{z_2 - z_1}$ is purely imaginary

(viii) Two points P (z_1) and Q(z_2) lie on the same side or opposite side of the line $\bar{a}z + a\bar{z} + b$ accordingly as $\bar{a}z_1 + a\bar{z}_1 + b$ and $\bar{a}z_2 + a\bar{z}_2 + b$ have same sign or opposite sign.

SOLVED EXAMPLES

Example – 1

Express the following in the form of a + ib, a, b $\in$ R, $i = \sqrt{-1}$. State the values of a and b.

(i) $\frac{i(4+3i)}{(1-i)}$ (ii) $\frac{(2+i)}{(3-i)(1+2i)}$

(iii) $\frac{4i^8 - 3i^9 + 3}{3i^{11} - 4i^{10} - 2}$ (iv) $(1+i)^6 + (1-i)^3$

Sol. (i) $z = \frac{i(4+3i)}{(1-i)}$

$$= \frac{4i + 3i^2}{1-i} \times \frac{1+i}{1+i}$$

$$= \frac{(4i-3)(1+i)}{1-i^2}$$

$$= \frac{4i + 4i^2 - 3 - 3i}{1+1}$$

$$= \frac{-7+i}{2}$$

$$= -\frac{7}{2} + \frac{1}{2}i$$

here, $a = -\frac{7}{2}$, $b = \frac{1}{2}$

(ii) $z = \frac{(2+i)}{(3-i)(1+2i)}$

$$= \frac{2+i}{3+6i-i-2i^2}$$

$$= \frac{2+i}{5+5i} \times \frac{5-5i}{5-5i}$$

$$= \frac{10 - 10i + 5i - 5i^2}{25 - 25i^2}$$

$$= \frac{15-5i}{50}$$

$$= \frac{3}{10} - \frac{1}{10}i$$

here, $a = \frac{3}{10}$, $b = -\frac{1}{10}$

(iii) $z = \frac{4i^8 - 3i^9 + 3}{3i^{11} - 4i^{10} - 2}$

$$= \frac{4.i^4.i^4 - 3i^4.i^4 i + 3}{3.i^4.i^4.i^3 - 4i^4.i^4.i^2 - 2}$$

$$= \frac{4 - 3i + 3}{-3i + 4 - 2}$$

$$= \frac{7-3i}{2-3i} \times \frac{2+3i}{2+3i}$$

$$= \frac{14 + 21i - 6i - 9i^2}{4 - 9i^2}$$

$$= \frac{23}{13} + \frac{15}{13}i$$

here, $a = \frac{23}{13}$, $b = \frac{15}{13}$

(iv) $(1+i)^6 = \{(1+i)^2\}^3 = (1+i^2+2i)^3 = (1-1+2i)^3 = 8i^3 = -8i$

and $(1-i)^3 = 1 - i^3 - 3i + 3i^2 = 1 + i - 3i - 3 = -2 - 2i$

Therefore, $(1+i)^6 + (1-i)^3 = -8i - 2 - 2i = -2 - 10i$

here, $a = -2$, $b = -10$

Example – 2

Express $\dfrac{1}{(1-\cos\theta)+2i\sin\theta}$ in the form A + iB.

Sol. *Now,* $\dfrac{1}{(1-\cos\theta)+2i\sin\theta}=\dfrac{1}{2\sin^2\frac{\theta}{2}+4i\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$

$$=\frac{1}{2\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2}+2i\cos\frac{\theta}{2}\right)}\times\frac{\sin\frac{\theta}{2}-2i\cos\frac{\theta}{2}}{\left(\sin\frac{\theta}{2}-2i\cos\frac{\theta}{2}\right)}$$

$$=\frac{\sin\frac{\theta}{2}-2i\cos\frac{\theta}{2}}{2\sin\frac{\theta}{2}\left(\sin^2\frac{\theta}{2}+4\cos^2\frac{\theta}{2}\right)}$$

$$=\frac{\sin\frac{\theta}{2}-2i\cos\frac{\theta}{2}}{2\sin\frac{\theta}{2}\left(1+3\cos^2\frac{\theta}{2}\right)}$$

$$\Rightarrow A+iB=\frac{1}{2\left(1+3\cos^2\frac{\theta}{2}\right)}-i\frac{\cot\frac{\theta}{2}}{1+3\cos^2\frac{\theta}{2}}$$

Example – 3

Find the real values of x and y for which the following equation is satisfied

$$\frac{(1+i)\,x-2i}{3+i}+\frac{(2-3i)\,y+i}{3-i}=i\,.$$

Sol. $\dfrac{(1+i)\,x-2i}{3+i}+\dfrac{(2-3i)\,y+i}{3-i}=i$

$\Rightarrow (1+i)(3-i)x-2i(3-i)+(3+i)(2-3i)y+i(3+i)=10i$

$\Rightarrow 4x+2ix-6i-2+9y-7iy+3i-1=10i$

$\Rightarrow 4x+9y-3=0$ and $2x-7y-3=10$

$\Rightarrow x=3$ and $y=-1$

Example – 4

Prove that : $x^4+4=(x+1+i)(x+1-i)(x-1+i)(x-1-i)$.

Sol. Consider R.H.S.

$=[(x+1+i)(x+1-i)][(x-1+i)(x-1-i)]$

$=[(x+1)^2-i^2][(x-1)^2-i^2)]$

$=(x^2+2x+1+1)(x^2-2x+1+1)$

$=[(x^2+2)+2x][(x^2+2)-2x]$

$=(x^2+2)^2-(2x)^2$

$=x^4+4x^2+4-4x^2=x^4+4=$ L.H.S.

Example – 5

Find the value of x^3+x^2-x+22 if $x=1+2i$

Sol. $x=1+2i$

$(x-1)^2=(2i)^2$

$x^2-2x+5\quad=0$

Now, $x^3+x^2-x+22=(x^2-2x+5)(x+3)+7$

So, Putting $x=1+2i$, we get :-

x^3+x^2-x+22

$=0+7=7$

Example – 6

Find the modulus and amplitude of the following complex numbers.

(i) $\sqrt{3}+\sqrt{2}i$ (ii) $1+i$

Sol. (i) $z=\sqrt{3}+\sqrt{2}i$ here $a=\sqrt{3},\ b=\sqrt{2}$

$$\therefore\ |z| = \sqrt{a^2+b^2} = \sqrt{3+2} = \sqrt{5}$$

$$\text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{3}}\right) = \tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$$

(ii) $z = 1+i$ here $a=1, b=1$

$$|z| = \sqrt{a^2+b^2} = \sqrt{1+1} = \sqrt{2}$$

$$\text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{1}{1}\right) = \tan^{-1}1 = \frac{\pi}{4}\ (a, b > 0 \text{ I quadrant})$$

Example – 7

$|z|\le 1, |w|\le 1$, show that

$|z-w|^2 \le (|z|-|w|)^2+(\arg z-\arg w)^2$

Sol. $|z-w|^2 = |z|^2+|w|^2-2|z||w|\cos(\arg z-\arg w)$

$= |z|^2+|w|^2-2|z||w|+2|z||w|-2|z||w|\cos(\arg z-\arg w)$

$$=(|z|-|w|)^2+2|z||w|.2\sin^2\left(\frac{\arg z-\arg w}{2}\right) \quad ...(i)$$

$$\therefore |z-w|^2 \le (|z|-|w|)^2+4.1.1\left(\frac{\arg z-\arg w}{2}\right)^2 \quad [\because \sin\theta\le\theta]$$

$$\Rightarrow |z-w|^2 \le (|z|-|w|)^2+(\arg z-\arg w)^2$$

Example – 8

For $z=2+3i$ verify the following :

(i) $z=\bar{\bar{z}}$ (ii) $z\bar{z}=|z|^2$

(ii) $(z+\bar{z})$ is real (iv) $z-\bar{z}$ is imaginary

Sol. $z=2+3i$

$\bar{z}=2-3i$

(i) $(\bar{\bar{z}}) = 2+3i$

Hence, $\bar{\bar{z}} = z$

(ii) $z.(\bar{z}) = (2+3i)(2-3i)$

$= 4-9i^2 = 13$

$|z| = \sqrt{4+9} = \sqrt{13}$

Hence, $z.\bar{z} = |z|^2$

(iii) $z+\bar{z} = 2+3i+2-3i$

$=4$

Hence, $z+\bar{z}$ is a real number.

(iv) $z-\bar{z} = (2+3i)-(2-3i)$

$= 2+3i-2+3i$

$= 6i$

Hence, $z-\bar{z}$ is an imaginary number.

Example – 9

If $x+iy=\sqrt{\dfrac{a+ib}{c+id}}$, prove that $\left(x^2+y^2\right)^2=\dfrac{a^2+b^2}{c^2+d^2}$

Sol. $x+iy = \sqrt{\dfrac{a+ib}{c+id}}$

$\therefore$ $x-iy = \sqrt{\dfrac{a-ib}{c-id}}$

(Taking complex conjugate)

$\therefore$ $(x+iy)(x-iy)=\sqrt{\dfrac{a+ib}{c+id}\times\dfrac{a-ib}{c-id}}$

$x^2-i^2y^2=\sqrt{\dfrac{a^2-i^2b^2}{c^2-i^2d^2}}$

$\therefore$ $x^2+y^2=\sqrt{\dfrac{a^2+b^2}{c^2+d^2}}$

$\therefore$ $(x^2+y^2)^2 = \dfrac{a^2+b^2}{c^2+d^2}$

Example – 10

(i) If $|z|=1$, prove that $\dfrac{z-1}{z+1}(z\neq-1)$ is a purely imaginary number.

(ii) If the number $\dfrac{z-1}{z+1}$ is purely imaginary, then prove that $|z|=1$.

Sol. (i) Let $w=\dfrac{z-1}{z+1}$

$$\operatorname{Re}(w)=\frac{w+\overline{w}}{2}=\frac{\left(\dfrac{z-1}{z+1}\right)+\left(\overline{\dfrac{z-1}{z+1}}\right)}{2}$$

$$=\frac{1}{2}\left[\frac{z-1}{z+1}+\frac{\overline{z}-1}{\overline{z}+1}\right]=\frac{1}{2}\left[\frac{z-1}{z+1}+\frac{\frac{1}{z}-1}{\frac{1}{z}+1}\right]$$

$$=\frac{1}{2}\left[\frac{z-1}{z+1}+\frac{1-z}{1+z}\right]=\frac{1}{2}\left[\frac{z-1}{z+1}-\frac{z-1}{z+1}\right]=0$$

$\Rightarrow$ $\operatorname{Re}(w)=0$

$\Rightarrow$ w is a purely imaginary number.

(ii) $w=\dfrac{z-1}{z+1}$

As w is purely imaginary.

$\operatorname{Re}(w)=0$

$$\Rightarrow \frac{w+\overline{w}}{2}=0 \Rightarrow \frac{\dfrac{z-1}{z+1}+\overline{\dfrac{z-1}{z+1}}}{2}=0$$

$$\Rightarrow \frac{z-1}{z+1}=-\frac{\overline{z}-1}{\overline{z}+1} \Rightarrow \frac{z-1}{z+1}=\frac{1-\overline{z}}{1+\overline{z}}$$

Apply componendo-dividendo to get :

$$z=\frac{1}{\overline{z}} \Rightarrow z\overline{z}=1 \Rightarrow |z|^2=1 \Rightarrow |z|=1.$$

Example – 11

If z_1 and z_2 are two complex numbers such that $|z_1|<1<|z_2|$, then prove that $\left|\dfrac{1-z_1\overline{z}_2}{z_1-z_2}\right|<1$.

Sol. Given, $|z_1|<1$ and $|z_2|>1$ ... (i)

Then, to prove

$$\left|\frac{1-z_1\overline{z}_2}{z_1-z_2}\right|<1$$

$$\Rightarrow |1-z_1\overline{z}_2|<|z_1-z_2| \ldots (ii) \quad \left[\text{using } \left|\frac{z_1}{z_2}\right|=\frac{|z_1|}{|z_2|}\right]$$

On squaring both sides, we get,

$$\left(1-z_1\overline{z}_2\right)\left(1-\overline{z}_1z_2\right)<\left(z_1-z_2\right)\left(\overline{z}_1-\overline{z}_2\right)\left[\text{using } |z|^2=z\overline{z}\right]$$

$$\Rightarrow 1-z_1\overline{z}_2-\overline{z}_1z_2+z_1\overline{z}_1z_2\overline{z}_2<z_1\overline{z}_1-z_1\overline{z}_2-z_2\overline{z}_1+z_2\overline{z}_2$$

$$\Rightarrow 1+|z_1|^2|z_2|^2<|z_1|^2+|z_2|^2$$

$$\Rightarrow 1-|z_1|^2-|z_2|^2+|z_1|^2|z_2|^2<0$$

$$\Rightarrow \left(1-|z_1|^2\right)\left(1-|z_2|^2\right)<0 \quad \ldots \text{(iii)}$$

Which is true by equation (i) as $|z_1|<1$ and $|z_2|>1$

$\therefore \left(1-|z_1|^2\right)>0$

and $\left(1-|z_2|^2\right)<0$

$\therefore$ Equation (iii) is true whenever equation (ii) is true.

$$\Rightarrow \left|\frac{1-z_1\overline{z}_2}{z_1-z_2}\right|<1$$

Example – 12

Express the following complex numbers in the polar form :

(i) $\frac{1+i}{1-i}$ (ii) $\frac{2+6\sqrt{3}i}{5+\sqrt{3}i}$

Sol. Let $z=\frac{1+i}{1-i}$, and, let r (cos θ + i sin θ) be the polar form of z.

Then, $r = |z|$ and $\theta = \arg(z)$.

Now, $z = \frac{1+i}{1-i} = \frac{(1+i)(1+i)}{(1-i)(1+i)}$

$$= \frac{1+2i+i^2}{1-i^2} = \frac{1+2i-1}{1+1} = i = 0+1\,i$$

$\therefore$ $r = |z| = \sqrt{0+1} = 1$.

Since the point (0, 1) representing z = 0 + i lies on positive direction of imaginary axis. Therefore,

$\arg(z) = \pi/2$.

Hence, $z = 1\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

(ii) Let $z = \frac{2+6\sqrt{3}i}{5+\sqrt{3}i}$, and let r (cos θ + i sin θ) be the polar form of z. Then, $r = |z|$ and $\theta = \arg(z)$

Now, $z = \frac{2+6\sqrt{3}i}{5+\sqrt{3}i}$

$\Rightarrow$ $z = \frac{2+6\sqrt{3}i}{5+\sqrt{3}i}\cdot\frac{(5-\sqrt{3}i)}{(5-\sqrt{3}i)}$

$$= \frac{28+28\sqrt{3}i}{28} = 1+i\sqrt{3}$$

$r = |z| = \sqrt{1+3} = 2$.

Let α be the smallest positive angle given by

$$\tan^{-1}\left(\frac{\mathrm{Im}(z)}{\mathrm{Re}(z)}\right) = \tan^{-1}\left(\sqrt{3}/1\right) = \tan^{-1}\left(\sqrt{3}\right).$$

Then, $\alpha = \frac{\pi}{3}$. Since the point $(1, \sqrt{3})$ lies in first quadrant.

$\therefore$ $\theta = \alpha = \frac{\pi}{3}$.

Hence, the polar form of z is $z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

Example – 13

Prove that there exists no complex number z such that

$|z| < \frac{1}{3}$ and $\sum_{r=1}^{n} a_r z^r = 1$, where $|a_r| < 2$.

Sol. Given, $a_1 z + a_2 z^2 + \ldots + a_n z^n = 1$

and $|z| < \frac{1}{3}$ ...(i)

$\therefore \left|a_1 z + a_2 z^2 + a_3 z^3 + \ldots + a_n z^n\right| = 1$

$\Rightarrow |a_1 z| + |a_2 z^2| + |a_3 z^3| + \ldots + |a_n z^n| \geq 1$

[using $|z_1 + z_2| \leq |z_1| + |z_2|$]

$\Rightarrow 2\{|z| + |z|^2 + |z|^3 + \ldots + |z|^n\} > 1$ [using $|a_r| < 2$]

$\Rightarrow \frac{2|z|(1-|z|^n)}{1-|z|} > 1$ [using sum of n terms of GP]

$\Rightarrow 2|z| - 2|z|^{n+1} > 1 - |z|$

$\Rightarrow 3|z| > 1 + 2|z|^{n+1} \Rightarrow |z| > \frac{1}{3} + \frac{2}{3}|z|^{n+1}$

$\Rightarrow |z| > \frac{1}{3}$, which contradicts ... (i)

$\therefore$ There exists no complex number z such that

$|z| < \frac{1}{3}$ and $\sum_{r=1}^{n} a_r z^r = 1$

Example – 14

If α and β are roots of $x^2 - 2x + 4 = 0$ then find $\alpha^n + \beta^n$.

Sol. $\alpha, \beta = 1 \pm i\sqrt{3}$

$$\alpha = 1 + i\sqrt{3}, \beta = 1 - i\sqrt{3}$$

$$\alpha^n = \left(1 + i\sqrt{3}\right)^n$$

$$= 2^n \left(\frac{1 + i\sqrt{3}}{2}\right)^n = 2^n\left(\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}\right)$$

$$and\ \beta^n = \left(1 - i\sqrt{3}\right)^n$$

$$= 2^n\left(\frac{1 - i\sqrt{3}}{2}\right)^n$$

$$= 2^n\left(\cos\frac{n\pi}{3} - i\sin\frac{n\pi}{3}\right)$$

$$\alpha^n + \beta^n = 2^n.2\cos\frac{n\pi}{3}$$

$$= 2^{n+1} \cos\left(\frac{n\pi}{3}\right)$$

Example – 15

It is given that n is an odd integer greater than 3, but not a multiple of 3. Prove that $x^3 + x^2 + x$ is a factor of $(x + 1)^n - x^n - 1$.

Sol. We have $x^3 + x^2 + x = x(x^2 + x + 1) = x(x - \omega)(x - \omega^2)$.

where ω, ω^2 are cube roots of unity but not equal to 1. Moreover, $\omega^3 = 1$.

$x^3 + x^2 + x$ is a factor of $(x + 1)^n - x^n - 1$. It means that $(x + 1)^n - x^n - 1$ should be zero at $x = 0, \omega, \omega^2$.

At $x = 0$,

$(x + 1)^n - x^n - 1 = 1^n - 1 = 1 - 1 = 0$

At $x = \omega$

$(x + 1)^n - x^n - 1 = (1 + \omega)^n - \omega^n - 1 = (-\omega^2)^n - \omega^n - 1$

$= (-1)^n \omega^{2n} - \omega^n - 1 = -[\omega^{2n} + \omega^n + 1] = 0$ as n is not a multiple of 3.

At $x = \omega^2$,

$(x + 1)^n - x^n - 1 = (1 + \omega^2)^n - \omega^{2n} - 1$

$= (-\omega)^n - \omega^{2n} - 1 = -[\omega^n + \omega^{2n} + 1] = 0$

$\Rightarrow$ $x^3 + x^2 + x$ is a factor of $(x + 1)^n - x^n - 1$.

Example – 16

Let a complex number α, $\alpha \neq 1$, be a root of the equation

$$z^{p+q} - z^p - z^q + 1 = 0$$

where p, q are distinct primes. Show that either

$$1 + \alpha + \alpha^2 + ... + \alpha^{p-1} = 0$$

or $\qquad 1 + \alpha + \alpha^2 + ... + \alpha^{q-1} = 0$

but not both together.

Sol. Given $z^{p+q} - z^p - z^q + 1 = 0$ ... (i)

$$\Rightarrow \left(z^p - 1\right)\left(z^p - 1\right) = 0$$

Since, α is root of equation (i), either

$$\alpha^p - 1 = 0\ or\ \alpha^q - 1 = 0$$

$$\Rightarrow Either\ \frac{\alpha^p - 1}{\alpha - 1} = 0\ or\ \frac{\alpha^q - 1}{\alpha - 1} = 0$$

$$\Rightarrow Either\ 1 + \alpha + \alpha^2 + ... + \alpha^{p-1} = 0$$

$$or\ 1 + \alpha + ... + \alpha^{q-1} = 0$$

But $\alpha^p - 1 = 0\ and\ \alpha^q - 1 = 0$ cannot occur simultaneously as p and q are distinct primes, so neither p divides q nor q divides p, which is the requirement for $1 = \alpha^p = \alpha^q$

Example – 17

Find the value of :

$$\sum_{r=1}^{r=6}\left[\sin\frac{2\pi r}{7} - i\cos\frac{2\pi r}{7}\right]$$

Sol. Let $S = \sum_{r=1}^{r=6}\left[1.\sin\frac{2\pi r}{7} - i\cos\frac{2\pi r}{7}\right]$

$$= \sum_{r=1}^{r=6}\left[-i^2 \sin\frac{2\pi r}{7} - i\cos\frac{2\pi r}{7}\right]$$

Take $(-i)$ common to get :

$$= -i\sum_{r=1}^{r=6}\left[\cos\frac{2\pi r}{7} + i\sin\frac{2\pi r}{7}\right]$$

$= -i$ (sum of 7th roots of unity $- 1$)

$= -i(0 - 1) = i$

Example – 18

If 1, a_1, a_2, ..., a_{n-1} are the n^{th} roots of unity, then show that $(1-a_1)(1-a_2)(1-a_3)\ldots(1-a_{n-1}) = n$.

Sol. Since, 1, a_1, a_2, ... a_{n-1} are nth roots of unity.

$$\Rightarrow \left(x^n-1\right)=\left(x-1\right)\left(x-a_1\right)\left(x-a_2\right)\ldots\left(x-a_{n-1}\right)$$

$$\Rightarrow \frac{x^n-1}{x-1}=\left(x-a_1\right)\left(x-a_2\right)\ldots\left(x-a_{n-1}\right)$$

$$\Rightarrow x^{n-1}+x^{n-2}+\ldots+x^2+x+1=\left(x-a_1\right)\left(x-a_2\right)\ldots\left(x-a_{n+1}\right)$$

$$\left[\because \frac{x^n-1}{x-1}=x^{n-1}+x^{n-2}+\ldots+x+1\right]$$

On putting x = 1, we get

$$n=\left(1-a_1\right)\left(1-a_2\right)\ldots\left(1-a_{n-1}\right)$$

Example – 19

Find the square roots of –15 – 8i.

Sol. Let $\sqrt{-15-8i}=x+iy$. Then,

$$\sqrt{-15-8i}=x+iy$$

$\Rightarrow$ $-15-8i=(x+iy)^2$

$\Rightarrow$ $-15-8i=(x^2-y^2)+2i\,xy$

$\Rightarrow$ $-15=x^2-y^2$ ...(i)

and, $2xy=-8$ ...(ii)

Now, $(x^2+y^2)^2=(x^2-y^2)^2+4x^2y^2$

$\Rightarrow$ $(x^2+y^2)^2=(-15)^2+64=289$

$\Rightarrow$ $x^2+y^2=17$ ...(iii)

On solving (i) and (iii), we get

$x^2 = 1$ and $y^2 = 16 \Rightarrow x = \pm 1$ and $y = \pm 4$

From (ii), 2xy is negative. So, x and y are of opposite signs.

$\therefore$ $(x = 1$ and $y = -4)$ or, $(x = -1$ and $y = 4)$

Hence, $\sqrt{-15-8i}=\pm\left(1-4i\right)$

Example – 20

Find the square root of i.

Sol. Let $\sqrt{i}=x+iy$. Then,

$$\sqrt{i}=x+iy$$

$\Rightarrow$ $i=(x+iy)^2$

$\Rightarrow$ $(x^2-y^2)+2i\,xy=0+i$

$\Rightarrow$ $x^2-y^2=0$ . ..(i)

and, $2xy=1$ ...(ii)

Now, $(x^2+y^2)^2=(x^2-y^2)^2+4x^2y^2$

$\Rightarrow$ $(x^2+y^2)^2=0+1=1$

$\Rightarrow$ $x^2+y^2=1$ $[\because x^2+y^2>0]$ ...(iii)

Solving (i) and (iii), we get

$x^2 = 1/2$ and $y^2 = 1/2$

$\Rightarrow$ $x=\pm 1/\sqrt{2}$ and $y=\pm 1/\sqrt{2}$

From (ii), we find that 2xy is positive. So, x and y are of same sign.

$$\therefore \left(x=\frac{1}{\sqrt{2}} \text{ and } y=\frac{1}{\sqrt{2}}\right) \text{ or } \left(x=-\frac{1}{\sqrt{2}} \text{ and } y=-\frac{1}{\sqrt{2}}\right)$$

Hence, $\sqrt{i}=\pm\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}i}\right)=\pm\frac{1}{\sqrt{2}}(1+i)$

Example – 21

Find all circles which are orthogonal to | z | = 1 and |z–1|=4.

Sol. $|z|=1; |z-1|=4$

$\Rightarrow$ $x^2+y^2=1$ and $(x-1)^2+y^2=4^2$

$S_1: x^2+y^2=1$; $S_2: x^2+y^2-2x-15=0$ and

$S: x^2+y^2+2gx+2fy+c=0$

Using the condition of orthogonality of S and S_1, we have

$2g(0)+2f(0)=c-1 \Rightarrow c=1$

Similarly, using the condition of orthogonality of S and S_2, we have

$2g(-1)+2f(0)=1-15 \Rightarrow g=7$

radius $=\sqrt{g^2+f^2-c}=\sqrt{48+f^2}$

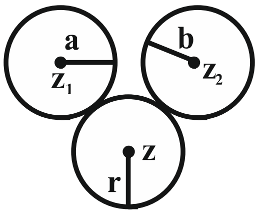
So, all the circles orthogonal to S_1 and S_2 are :

$$\left|z-(-7-if)\right|=\sqrt{48+f^2};\ f\in R$$

Example – 22

Prove that the locus of the centre of a circle which touches the circle $|z - z_1| = a$ and $|z - z_2| = b$ externally (z, z_1 and z_2 complex numbers) is a hyperbola.

Sol.



$|z - z_1| = a + r$

$|z - z_2| = b + r$

$\Rightarrow \left||z - z_1| - |z - z_2|\right| = |a - b| =$ constant

$\Rightarrow$ Locus of z is hyperbola.

Example – 23

If $\omega = \dfrac{z}{z - (1/3)i}$ and $|\omega| = 1$, then prove that z lies on a straight line

Sol. As given

$$w = \frac{z}{z - \frac{i}{3}} \Rightarrow |w| = \frac{|z|}{\left|z - \frac{i}{3}\right|} = 1$$

$\Rightarrow$ distance of z from origin and point $\left(0, \dfrac{1}{3}\right)$ is same,

hence z lies on the bisector of line joining the points (0,0) and $\left(0, \dfrac{1}{3}\right)$.

Hence z lies on straight line.

Example – 24

What is the locus of z, if amplitude of $z - 2 - 3i$ is $\dfrac{\pi}{4}$?

Sol. Amplitude $(z - z_0) = \theta$ represents a ray starting from z_0 and making an angle θ with positive real axis. So locus of z is a ray starting at $2 + 3i$ and making an angle $\dfrac{\pi}{4}$ with positive real axis.

Example – 25

Find the locus of point z satisfying the condition $\left|\dfrac{z - i}{z + i}\right| \geq 2$.

Sol. $\left|\dfrac{z - i}{z + i}\right|^2 \geq 4 \Rightarrow |z - i|^2 \geq 4|z + i|^2$

$\Rightarrow (z - i)(\bar{z} + i) \geq 4(z + i)(\bar{z} - i)$

$\Rightarrow z\bar{z} - i(\bar{z} - z) + 1 \geq 4z\bar{z} + 4i(\bar{z} - z) + 4$

$\Rightarrow 3|z|^2 + 5i(\bar{z} - z) + 3 \leq 0$

$\Rightarrow 3x^2 + 3y^2 + 10y + 3 \leq 0.$

Which represents the interior and boundary of the circle.

Example – 26

Show that the representive points of the complex numbers $i, -2 - 5i, 1 + 4i$ and $3 + 10i$ are collinear.

Sol. Let Cartesian coordinates of these points be A (0, 1), B (–2, – 5), C (1, 4) and D (3, 10)

$$y - 1 = \frac{-5 - 1}{-2 - 0}(x - 0)$$

$\Rightarrow y = 3x + 1$ ...(1)

Points C (1, 4) and D (3, 10) satisfy the equatioin (1). Hence points A, B, C and D are collinear.

Example – 27

Locate the complex number $z = x + iy$ for which

(i) $z^2 + \bar{z}^2 + 2|z|^2 < 8i(\bar{z} - z)$

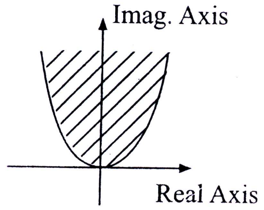
(ii) $\log_{1/3}\{\log_{1/2}(|z|^2 + 4|z| + 3)\} < 0$

Sol. (i) $z^2 + \bar{z}^2 + 2|z|^2 < 8i(\bar{z} - z)$

Substitute $z = x + iy$

$x^2 - y^2 + i(2xy) + x^2 - y^2 - i(2xy) + 2(x^2 + y^2) < 8i(-2iy)$

$\Rightarrow$ $4x^2 < 16y$ $\Rightarrow x^2 < 4y$



So, locus is the interior of the parabola $y = \dfrac{x^2}{4}$

(ii) $\log_{1/3}\{\log_{1/2}(|z|^2 + 4|z| + 3)\} < 0$

$\Rightarrow$ $\log_{1/2}(|z|^2 + 4|z| + 3) > 1$

$\Rightarrow$ $|z|^2 + 4|z| + 3 < \dfrac{1}{2}$

$\Rightarrow$ $|z|^2 + 4|z| + \dfrac{5}{2} < 0$

$\Rightarrow$ $|z| \in \left(\dfrac{-4-\sqrt{6}}{2}, \dfrac{-4+\sqrt{6}}{2}\right)$; But $|z| \geq 0$

$\Rightarrow$ So, no such value of z exists.

Example – 28

If $z \neq 1$ and $\dfrac{z^2}{z-1}$ is real, then prove that the point represented by the complex number z lies either on the real axis or on a circle passing through the origin

Sol. $\dfrac{z^2}{z-1} = \dfrac{x^2 - y^2 + 2ixy}{(x-1) + iy}$ *is real*

$\Rightarrow -(x^2 - y^2)y + 2xy(x-1) = 0$

$\Rightarrow y\left[-x^2 + y^2 + 2x^2 - 2x\right] = 0$

$\Rightarrow y = 0$ *or* $x^2 + y^2 - 2x = 0$

$\Rightarrow$ either real axis or circle passing through origin.

Example – 29

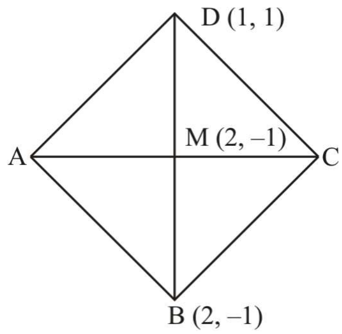
ABCD is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy BD = 2AC. Its points D and M represent the complex numbers $1 + i$ and $2 - i$ respectively. Find the complex number represented by A.

Sol. Let A be (x, y)

It is given that BD = 2AC $\Rightarrow$ MD = 2AM

Also DM is perpendicular to AM

$\Rightarrow (1-2)^2 + (1+1)^2 = 4[(x-2)^2 + (y+1)^2]$ ...(1)



and $\dfrac{y+1}{x-2} \cdot \dfrac{1+1}{1-2} = -1 \Rightarrow 2(y+1) = x-2$

With $x - 2 = 2(y+1)$, (1) gives $(y+1)^2 = 1/4$

$\Rightarrow$ $y = -1/2, -3/2$ $\Rightarrow$ $x = 3, 1$

$\Rightarrow$ A represent $z = 3 - i/2$, or $1 - 3i/2$

Alternative Solution :

MD = 2AM and AM $\perp$ DM i.e. angle AMD = $\pi/2$

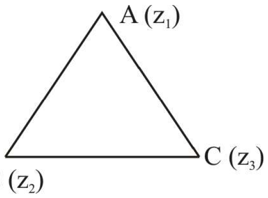
$\Rightarrow \dfrac{z-(2-i)}{(1+i)-(2-i)} = \dfrac{AM}{MD} . e^{\pm\frac{i\pi}{2}} = \pm i/2$

$\Rightarrow z - (2-i) = \pm\dfrac{i}{2}(-1+2i) \Rightarrow z = 3 - i/2$ or $1 - 3i/2$

Example – 30

Show that the triangle whose vertices are the points represented by the complex numbers z_1, z_2, z_3 on the argand diagram is equilateral if and only if $z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$.

Sol.



Applying rotation about B,

$$\frac{z_1 - z_2}{z_3 - z_2} = e^{i\pi/3} \quad ...(1)$$

Applying rotation about C,

$$\frac{z_2 - z_3}{z_1 - z_3} = e^{i\pi/3} \quad ...(2)$$

From (1) and (2), $\frac{z_1 - z_2}{z_3 - z_2} = \frac{z_2 - z_3}{z_1 - z_3}$

Simplifying, we get the required Conditions

Example – 31

Complex numbers z_1, z_2, z_3 are the vertices A, B, C respectively of an isosceles right angled triangle with right angle at C. Show that $(z_1 - z_2)^2 = 2(z_1 - z_3)(z_3 - z_2)$.

Sol. Since, triangle is a right angled isosceles triangle. So, rotating z_1 and z_3 in anti-clockwise direction through an angle of $\frac{\pi}{2}$, *we get* z_2

$$\frac{z_2 - z_3}{z_1 - z_3} = \frac{|z_2 - z_3|}{|z_1 - z_3|} e^{i\frac{\pi}{2}}$$

where, $|z_2 - z_3| = |z_1 - z_3|$

$$\Rightarrow (z_2 - z_3) = i(z_1 - z_3)$$

On squaring both sides, we get

$$(z_2 - z_3)^2 = -(z_1 - z_3)^2$$

$$\Rightarrow z_2^2 + z_3^2 - 2z_2z_3 = -z_1^2 - z_3^2 + 2z_1z_3$$

$$\Rightarrow z_1^2 + z_2^2 - 2z_1z_2 = 2z_1z_3 + 2z_2z_3 - 2z_3^2 - 2z_1z_2$$

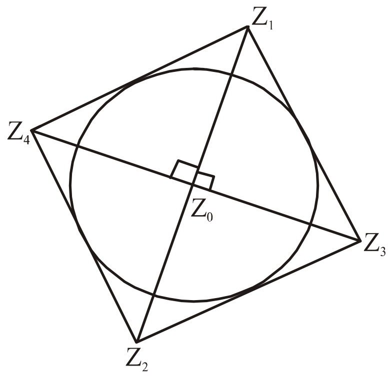
$$\Rightarrow (z_1 - z_2)^2 = 2\{(z_1z_3 - z_3^2) + (z_2z_3 - z_1z_2)\}$$

$$\Rightarrow (z_1 - z_2)^2 = 2(z_1 - z_3)(z_3 - z_2)$$

Example – 32

If one of the vertices of the square circumscribing the circle $|z - 1| = \sqrt{2}$ is $2 + \sqrt{3}\,i$. Find the other vertices of square.

Sol. Here, centre of circle is (1, 0) is also the mid-point of diagonals of square



$$\Rightarrow \frac{z_1 + z_2}{2} = z_0$$

$$\Rightarrow z_2 = -\sqrt{3}\,i \; [where,\ z_0 = 1 + 0i]$$

$$and\ \frac{z_3 - 1}{z_1 - 1} = e^{\pm\frac{i\pi}{2}}$$

$$\Rightarrow z_3 = 1 + (1 + \sqrt{3}i).\left(\cos\frac{\pi}{2} \pm \sin\frac{\pi}{2}\right)\left[\because z_1 = 2 + \sqrt{3}i\right]$$

$$\Rightarrow z_3 = 1 \pm i(1 + \sqrt{3}i) = (1 \mp \sqrt{3}) \pm i = (1 - \sqrt{3}) + i \ and\ z_4 = (1 + \sqrt{3}) - i$$

Example – 33

Let the complex number z_1, z_2 and z_3 be the vertices of an equilateral triangle. Let z_0 be the circumcentre of the triangle. Then prove that $z_1^2+z_2^2+z_3^2=3z_0^2$.

Sol. Since z_1, z_2, z_3 are the vertices of an equilateral triangle.

$$\therefore Circumcentre(z_0)=Centroid=\left(\frac{z_1+z_2+z_3}{3}\right) \quad \text{... (i)}$$

Also, for equilateral triangle

$$z_1^2+z_2^2+z_3^2=z_1z_2+z_2z_3+z_3z_1 \quad \text{... (ii)}$$

On squaring Equation (i), we get

$$9z_0^2=z_1^2+z_2^2+z_3^2+2(z_1z_2+z_2z_3+z_3z_1)$$

$$\Rightarrow 9z_0^2=z_1^2+z_2^2+z_3^2+2(z_1^2+z_1^2+z_3^2) \text{ [from equation(ii)]}$$

$$\Rightarrow 3z_0^2=z_1^2+z_2^2+z_3^2$$

Example – 34

Find the centre and radius of the circle formed by all the points represented by z = x + iy satisfying the relation $\left|\frac{z-\alpha}{z-\beta}\right|=k\ (k\neq 1)$, where α and β are the constant complex numbers given by $\alpha=\alpha_1+i\alpha_2$, $\beta=\beta_1+i\beta_2$.

Sol. As we know, $|z|^2=z.\bar{z}$

Given, $\frac{|z-\alpha|^2}{|z-\beta|^2}=k^2$

$$\Rightarrow (z-\alpha)(\bar{z}-\bar{\alpha})=k^2(z-\beta)(\bar{z}-\bar{\beta})$$

$$\Rightarrow |z|^2-\alpha\bar{z}-\bar{\alpha}z+|\alpha|^2=k^2\left(|z|^2-\beta\bar{z}-\bar{\beta}z+|\beta|^2\right)$$

$$\Rightarrow |z|^2(1-k^2)-(\alpha-k^2\beta)\bar{z}-(\bar{\alpha}-\bar{\beta}k^2)z+(|\alpha|^2-k^2|\beta|^2)=0$$

$$\Rightarrow |z|^2-\frac{(\alpha-k^2\beta)}{(1-k^2)}\bar{z}-\frac{(\bar{\alpha}-\bar{\beta}k^2)}{(1-k^2)}z+\frac{|\alpha|^2-k^2|\beta|^2}{(1-k^2)}=0 \ldots(i)$$

On comparing with equation of circle.

$$|z|^2+a\bar{z}+\bar{a}z+b=0$$

whose centre is (-a) and radius $=\sqrt{|a|^2-b}$

$$\therefore \text{Centre for equation. (i)} = \frac{\alpha-k^2\beta}{1-k^2}$$

and radius $=\sqrt{\left(\frac{\alpha-k^2\beta}{1-k^2}\right)\left(\frac{\bar{\alpha}-k^2\bar{\beta}}{1-k^2}\right)-\frac{\alpha\bar{\alpha}-k^2\beta\bar{\beta}}{1-k^2}}$

$$=\left|\frac{k(\alpha-\beta)}{1-k^2}\right|$$

Example – 35

Let z_1, z_2, z_3 be three distinct complex numbers satisfying $|z_1-1|=|z_2-1|=|z_3-1|$. Let A, B, and C be the points represented in the argand plane corresponding to z_1, z_2 and z_3 respectively. Prove that $z_1+z_2+z_3=3$ if and only of Δ ABC is an equilateral triangle.

Sol. $|z_1-1|=|z_2-1|=|z_3-1|$

$\Rightarrow$ The point corresponding to 1 (say P) is equidistant from the points A, B and C.

$\Rightarrow$ P is the circumcentre of the Δ ABC

Now if $z_1+z_2+z_3=3$ then the point corresponding to centroid of the ΔABC is $\frac{z_1+z_2+z_3}{3}=1$

$\Rightarrow$ circumcentre and centroid coincide $\Rightarrow$ Δ ABC is equilateral

Conversely if Δ ABC is equilateral, then centroid is the same as the circumcentre i.e. P. Hence centroid

$$\frac{z_1+z_2+z_3}{3}=1\Rightarrow z_1+z_2+z_3=3$$

Example – 36

If A and B represent the complex number z_1 and z_2 such that $|z_1 + z_2| = |z_1 - z_2|$, then find the circumcentre of triangle OAB where O is the origin.

Sol. $|z_1 + z_2|^2 = |z_1 - z_2|^2 \Rightarrow (z_1 + z_2)(\bar{z}_1 + \bar{z}_2) = (z_1 - z_2)(\bar{z}_1 - \bar{z}_2)$

$$\Rightarrow z_2\bar{z}_1 + z_1\bar{z}_2 = 0$$

$$\Rightarrow \frac{z_1}{z_2} + \frac{\bar{z}_1}{\bar{z}_2} = 0$$

$\Rightarrow \frac{z_1}{z_2}$ is purely imaginary

$$\Rightarrow \arg\left(\frac{z_1}{z_2}\right) = \pm\frac{\pi}{2}$$

$\Rightarrow$ OAB is a right angled triangle right angled at O.

$\Rightarrow$ circumcentre is $\frac{z_1 + z_2}{2}$

Example – 37

Find all non-zero complex numbers z satisfying $\bar{z} = iz^2$.

Sol. Let $z = x + iy$

Given $\bar{z} = iz^2$

$$\Rightarrow \left(\overline{x + iy}\right) = i\,(x + iy)^2$$

$$\Rightarrow x - iy = i\left(x^2 - y^2 + 2ixy\right)$$

$$\Rightarrow x - iy = -2xy + i\left(x^2 - y^2\right)$$

On equating real and imaginary parts, we get

$x = -2xy$ and $-y = x^2 - y^2$

$$\Rightarrow x + 2xy = 0 \text{ and } x^2 - y^2 + y = 0$$

$$\Rightarrow x(1 + 2y) = 0$$

$$\Rightarrow x = 0 \text{ or } y = -\frac{1}{2}$$

When $x = 0$, $x^2 - y^2 + y = 0$

$$\Rightarrow 0 - y^2 + y = 0$$

$$\Rightarrow y(1 - y) = 0$$

$$\Rightarrow y = 0 \text{ or } y = 1$$

When, $y = -\frac{1}{2}$, $x^2 - y^2 + y = 0$

$$\Rightarrow x^2 - \frac{1}{4} - \frac{1}{2} = 0 \Rightarrow x^2 = \frac{3}{4}$$

$$\Rightarrow x = \pm\frac{\sqrt{3}}{2}$$

Therefore, $z = 0 + i0,\ 0 + i;\ \pm\frac{\sqrt{3}}{2} - \frac{i}{2}$

$$z = i, \pm\frac{\sqrt{3}}{2} - \frac{i}{2}$$

Example – 38

Solve the equation $z^2 = \bar{z}$, where $z = x + iy$

Sol. $z^2 = \bar{z} \Rightarrow x^2 - y^2 + i2xy = x - iy$

Therefore, $x^2 - y^2 = x$ ... (1)

and $2xy = -y$ ... (2)

From (2), we have $y = 0$ or $x = -\frac{1}{2}$

When $y = 0$, from (1), we get $x^2 - x = 0$, i.e., $x = 0$ or $x = 1$.

When $x = -\frac{1}{2}$, from (1), we get $y^2 = \frac{1}{4} + \frac{1}{2}$ or

$y^2 = \frac{3}{4}$, i.e., $y = \pm\frac{\sqrt{3}}{2}$

Hence, the solutions of the given equations are

$$0 + i0, 1 + i0, -\frac{1}{2} + i\frac{\sqrt{3}}{2}, -\frac{1}{2} - i\frac{\sqrt{3}}{2}$$

Example – 39

If $iz^3 + z^2 - z + i = 0$, then show $|z| = 1$.

Sol. $iz^3 + z^2 - z + i = 0$

By substituting $z = i$ in the equation, we get $0 = 0$

Hence $z - i$ is a factor of $iz^3 + z^2 - z + i$

$\Rightarrow iz^2(z-i) - 1(z-i) = 0 \Rightarrow (iz^2 - 1)(z-i) = 0$

Either $iz^2 - 1 = 0$ or $z - i = 0$

When $z - i = 0$, $z = i$ $\therefore |z| = |0 + i.1| = \sqrt{0^2 + 1^2} = 1$.

When $iz^2 - 1 = 0$, $z^2 = 1/i = -i$

$\therefore |z^2| = |0 - 1.i| = \sqrt{0^2 + (-1)^2} = 1 \Rightarrow |z^2| = 1$ or $|z| = 1$

$\therefore$ In any case we have $|z| = 1$

Example – 40

Find all complex numbers z which satisfy the following equations :

(i) $z = \overline{z}$ (ii) $z = -\overline{z}$

(iii) $\overline{z} = 4 - z$ (iv) $z^2 = -\overline{z}$

Sol. Let $z = x + iy$. Then $\overline{z} = x - iy$.

(i) The equation $z = \overline{z}$ becomes $x + iy = x - iy$.

or $2iy = 0$ which gives $y = 0$.

Hence $z = x$ i.e., all the real numbers constitute the solutions of the given equation.

(ii) The equation $z = -\overline{z}$ is equivalent to

$x + iy = -(x - iy)$ or $2x = 0$ or $x = 0$.

Hence $z = iy$ i.e., the solutions of the given equation are all pure imaginary numbers.

(iii) $\overline{z} = 4 - z$ or $x - iy = 4 - x - iy$ or $x = 4 - x$.

This gives $x = 2$.

Hence $z = 2 + iy$.

$\therefore$ The given equation is satisfied by all complex numbers whose real part is 2.

(iv) $z^2 = -\overline{z}$ or $(x + iy)^2 = -(x - iy)$

or $x^2 - y^2 + 2ixy = -x + iy$

Equating real and imaginary parts, we get

$x^2 - y^2 = -x$ ...(i)

and $2xy = y$ or $y(2x - 1) = 0$ ...(ii)

From (ii) either $y = 0$ or $2x - 1 = 0$

i.e., $x = 1/2$.

When $y = 0$, (i) gives $x^2 = -x$ or $x(x + 1) = 0$ which gives $x = 0, -1$.

Thus we get two sets of solution $x = 0, y = 0$ and $x = -1, y = 0$.

When $x = 1/2$, (i) gives $y^2 = 3/4$ which gives

$$y = \pm\frac{\sqrt{3}}{2}.$$

Thus we get two more sets of solutions

$$x = \frac{1}{2}, y = \frac{\sqrt{3}}{2}, \text{ and } x = \frac{1}{2}, y = -\frac{\sqrt{3}}{2}.$$

Hence the given equation has in all the following four solutions : $z_1 = 0, z_2 = -1$,

$$z_3 = \frac{1}{2} + i\left(\frac{\sqrt{3}}{2}\right), z_4 = \frac{1}{2} - i\left(\frac{\sqrt{3}}{2}\right)$$

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Iota & powers of iota

1. $(\sqrt{-2})(\sqrt{-3})$ is equal to

(a) $\sqrt{6}$ (b) $-\sqrt{6}$

(c) $i\sqrt{6}$ (d) none of these

2. $(1+i)^4 \times \left(1+\frac{1}{i}\right)^4 =$

(a) 16 (b) 0

(c) 8 (d) 64

3. The value of $(1+i)(1+i^2)(1+i^3)(1+i^4)$ is

(a) 2 (b) 0

(c) 1 (d) i

4. The value of $1+i^2+i^4+i^6+...+i^{2n}$ is :

(a) positive (b) negative

(c) zero (d) cannot be determined

5. The value of sum $\sum_{n=1}^{13}\left(i^n+i^{n+1}\right)$, where $i=\sqrt{-1}$ equals

(a) i (b) $i-1$

(c) $-i$ (d) 0

6. The least positive integer n such that $\left(\frac{2i}{1+i}\right)^n$ is a positive integer, is

(a) 16 (b) 8

(c) 4 (d) 2

7. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then

(a) $x = 2n$, where n is any positive integer

(b) $x = 4n + 1$, where n is any positive integer

(c) $x = 2n + 1$, where n is any positive integer

(d) $x = 4n$, where n is any positive integer

8. For positive integers n_1, n_2 the value of expression $(1+i)^{n_1}+\left(1+i^3\right)^{n_1}+\left(1+i^5\right)^{n_2}+\left(1+i^7\right)^{n_2}$, here $i=\sqrt{-1}$ is a real number, if and only if

(a) $n_1 = n_2 + 1$ (b) $n_1 = n_2 - 1$

(c) $n_1 = n_2$ (d) $n_1 > 0, n_2 > 0$

9. If $\sqrt{x+iy} = \pm(a+ib)$, then $\sqrt{-x-iy}$ is equal to

(a) $\pm(b+ia)$ (b) $\pm(a-ib)$

(c) $(ai+b)$ (d) $\pm(b-ia)$

Algebra of complex number

10. The roots of the equation $x^4 - 1 = 0$, are :

(a) $1, 1, i, -i$ (b) $1, -1, i, -i$

(c) $1, -1, \omega, \omega^2$ (d) none of these

11. Inequality $a + ib > c + id$ can be explained only when :

(a) $b = 0, c = 0$ (b) $b = 0, d = 0$

(c) $a = 0, c = 0$ (d) $a = 0, d = 0$

12. If $a^2 + b^2 = 1$, then $\frac{1+b+ia}{1+b-ia}$ is equal to :

(a) 1 (b) 2

(c) $b + ia$ (d) $a + ib$

13. Let $z \neq -i$ be any complex number such that $\frac{z-i}{z+i}$ is a purely imaginary number then $z+\frac{1}{z}$ is:

(a) 0

(b) any non-zero real number other than 1.

(c) any non-zero real number.

(d) a purely imaginary number.

14. If $\sqrt{a+ib} = x + iy$, then possible value of $\sqrt{a-ib}$ is

(a) x^2+y^2 (b) $\sqrt{x^2+y^2}$

(c) $x+iy$ (d) $x-iy$

15. Square root of $5 + 12i$ is

(a) $\pm(3+2i)$ (b) $\pm(3-2i)$

(c) $2+4i$ (d) $-1-2i$

Modulus of Complex Number

16. The modulus of $\frac{1-i}{3+i}+\frac{4i}{5}$ is

(a) $\sqrt{5}$ unit (b) $\frac{\sqrt{11}}{5}$ unit

(c) $\frac{\sqrt{5}}{5}$ unit (d) $\frac{\sqrt{12}}{5}$ unit

17. If $(x+iy)=\sqrt{\frac{1+2i}{3+4i}}$, then $(x^2+y^2)^2$ is equal to

(a) 5 (b) 1/5

(c) 2/5 (d) 5/2

18. If $x+iy=(1+i)(1+2i)(1+3i)$, then $x^2+y^2=$

(a) 0 (b) 1

(c) 100 (d) none of these

Argument of Complex Number

19. The amplitude of $\sin\frac{\pi}{5}+i\left(1-\cos\frac{\pi}{5}\right)$ is

(a) $\frac{2\pi}{5}$ (b) $\frac{\pi}{15}$

(c) $\frac{\pi}{10}$ (d) $\frac{\pi}{5}$

20. If $z=\frac{1+2i}{1-(1-i)^2}$, then arg (z) equals

(a) 0 (b) $\frac{\pi}{2}$

(c) π (d) none of these

21. The magnitude and amplitude of $\frac{(1+i\sqrt{3})(2+2i)}{(\sqrt{3}-i)}$ are respectively

(a) $2,\frac{3\pi}{4}$ (b) $2\sqrt{2},\frac{3\pi}{4}$

(c) $2\sqrt{2},\frac{\pi}{4}$ (d) $2\sqrt{2},\frac{\pi}{2}$

22. If arg (z) = θ, then arg ($\bar{z}$) is equal to

(a) $\theta-\pi$ (b) $\pi-\theta$

(c) θ (d) $-\theta$

23. Let z, w be complex numbers such that $\bar{z}+i\bar{w}=0$ and arg (zw) = π. Then, arg (z) equals

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{4}$ (d) $\frac{5\pi}{4}$

24. If z and w are two non-zero complex numbers such that $|zw|=1$ and $\arg(z)-\arg(w)=\frac{\pi}{2}$, then $\bar{z}$ w is equal to

(a) 1 (b) –1

(c) i (d) –i

Polar/Euler's form of complex number

25. The value of $\frac{4(\cos 75^\circ+i\sin 75^\circ)}{0.4(\cos 30^\circ+i\sin 30^\circ)}$ is :

(a) $\frac{\sqrt{2}}{10}(1+i)$ (b) $\frac{\sqrt{2}}{10}(1-i)$

(c) $\frac{10}{\sqrt{2}}(1-i)$ (d) $\frac{10}{\sqrt{2}}(1+i)$

26. The principal amplitude of $(\sin 40^\circ+i\cos 40^\circ)^5$ is

(a) 70° (b) -110°

(c) 110° (d) -70°

27. Let $z=\frac{\cos\theta+i\sin\theta}{\cos\theta-i\sin\theta},\frac{\pi}{4}<\theta<\frac{\pi}{2}$. Then, arg (z) is

(a) 2θ (b) $2\theta-\pi$

(c) $\pi+2\theta$ (d) None of these

28. If $e^{i\theta}=\cos\theta+i\sin\theta$, then for the ΔABC, $e^{iA}\cdot e^{iB}\cdot e^{iC}$ is

(a) – i (b) 1

(c) – 1 (d) None of these

29. If $z_1=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ and $z_2=\sqrt{3}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$, then $|z_1 z_2|$ is

(a) 6 (b) $\sqrt{2}$

(c) $\sqrt{6}$ (d) $\sqrt{3}$

30. The polar form of $(i^{25})^3$ is

(a) $\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$ (b) $\cos\pi+i\sin\pi$

(c) $\cos\pi-i\sin\pi$ (d) $\cos\frac{\pi}{2}-i\sin\frac{\pi}{2}$

Properties of complex number

31. For any two complex numbers z_1 and z_2 and any real numbers a and b; $|(az_1-bz_2)|^2+|(bz_1+az_2)|^2$ is equal to :

(a) $(a^2+b^2)(|z_1|+|z_2|)$ (b) $(a^2+b^2)(|z_1|^2+|z_2|^2)$

(c) $(a^2+b^2)(|z_1|^2-|z_2|^2)$ (d) none of the above

32. If z is a non-zero complex number, then $\left|\frac{|\bar{z}|^2}{z\bar{z}}\right|$ is equal to

(a) $\left|\frac{\bar{z}}{z}\right|$ (b) $|z|$

(c) $|\bar{z}|$ (d) none of these

33. If $|z_1|=|z_2|=\ldots=|z_n|=1$, then the value of $|z_1+z_2+z_3+\ldots+z_n|$ is:

(a) 1 (b) $|z_1|+|z_2|+\ldots+|z_n|$

(c) $\left|\frac{1}{z_1}+\frac{1}{z_2}+\ldots+\frac{1}{z_n}\right|$ (d) none of these

34. If z_1, z_2, z_3 are complex numbers such that

$|z_1|=|z_2|=|z_3|=\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|=1$, then $|z_1+z_2+z_3|$ is

(a) equal to 1 (b) less than 1

(c) greater than 3 (d) equal to 3

Cube roots of unity

35. If the cube roots of unity are 1, ω, ω^2, then the roots of the equation $(x-1)^3+8=0$, are:

(a) $-1, 1+2\omega, 1+2\omega^2$ (b) $-1, 1-2\omega, 1-2\omega^2$

(c) $-1,-1,-1$ (d) $-1,-1+2\omega,-1-2\omega^2$

36. If $i=\sqrt{-1}$, then

$$4+5\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{334}-3\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{365}$$

is equal to

(a) $1-i\sqrt{3}$ (b) $-1+i\sqrt{3}$

(c) $4\sqrt{3}i$ (d) $-i\sqrt{3}$

37. If ω is a non-real cube root of unity, then the expression $(1-\omega)(1-\omega^2)(1+\omega^4)(1+\omega^8)$ is equal to

(a) 0 (b) 3

(c) 1 (d) 2

38. If $z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$, then

(a) $\text{Re}(z)=0$

(b) $\text{Im}(z)=0$

(c) $\text{Re}(z)>0, \text{Im}(z)>0$

(d) $\text{Re}(z)>0, \text{Im}(z)<0$

39. The value of amp (iω) + amp ($i\omega^2$), where $i=\sqrt{-1}$ and $\omega=\sqrt[3]{1}$ = non-real, is

(a) 0 (b) $\frac{\pi}{2}$

(c) π (d) None of these

40. If ω is an imaginary cube root of unity, then $(1+w-w^2)^7$ is equal to

(a) 128ω (b) −128ω

(c) $128\omega^2$ (d) $-128\omega^2$

41. Suppose z_1, z_2, z_3 are the vertices of an equilateral triangle inscribed in the circle $|z|=2$. If $z_1=1+i\sqrt{3}$, then z_2 and z_3 are

(a) $z_2=-2$ *and* $z_3=1-i\sqrt{3}$

(b) $z_2=-1$ *and* $z_3=-i\sqrt{3}$

(c) $z_2=1$ *and* $z_3=1-i\sqrt{3}$

(d) $z_2=-1$ *and* $z_3=1-i\sqrt{3}$

42. If $x = a + b$, $y = a\alpha + b\beta$, $z = a\beta + b\alpha$, where α, β are complex cube roots of unity, then xyz equals

(a) $a^2 + b^2$ (b) 0

(c) $a^3 + b^3$ (d) none of these

Geometrical interpretation of modulus

43. The complex numbers $z = x + iy$ which satisfy the equation

$\left|\frac{z-5i}{z+5i}\right| = 1$, lie on

(a) the x-axis

(b) the straight line $y = 5$

(c) a circle passing through the origin

(d) None of these

44. The inequality $|z - 4| < |z - 2|$ represents the region given by

(a) $\text{Re}(z) > 3$ (b) $\text{Re}(z) < 3$

(c) $\text{Re}(z) \geq 3$ (d) None of these

45. If $z = x + iy$ and $w = (1 - iz) / (z - i)$, then $|w| = 1$ implies that, in the complex plane

(a) z lies on the imaginary axis

(b) z lies on the real axis

(c) z lies on the unit circle

(d) None of these

46. If P represents the variable complex number z and $\text{Re}(z + 1/z + i) = 1$, then the locus of P is

(a) $x - y - 1 = 0$ (b) $x - y + 1 = 0$

(c) $2x - y - 1 = 0$ (d) $x - 2y - 1 = 0$

47. P represents the variable complex number z and $|2z - 3| = 2$, then the locus of P is

(a) $x^2 + y^2 - 12x + 5 = 0$ (b) $4x^2 + 4y^2 - 12x + 5 = 0$

(c) $4x^2 - 4y^2 - 12x + 5 = 0$ (d) $x^2 + 4y^2 - 12x = 0$

48. If complex number $z = x + iy$ satisfies the equation $\text{Re}(z + 1) = |z - 1|$, then z lies on

(a) $y = x$ (b) $y^2 = 4x$

(c) $y = 2x$ (d) $2y = x$

49. The equation $\bar{b}z + b\bar{z} = c$ where b is a non zero complex constant and c is real, represents

(a) A circle (b) A straight line

(c) A parabola (d) None of these

50. The equation $z\bar{z} + (2-3i)\, z + (2+3i)\bar{z} + 4 = 0$ represents a circle of radius

(a) 2 (b) 3

(c) 4 (d) 6

51. For real parameter t, the locus of the complex number $z = \left(1-t^2\right) + i\sqrt{1+t^2}$ in the complex plane is

(a) an ellipse (b) a parabola

(c) a circle (d) a hyperbola

52. The points z_1, z_2, z_3, z_4 in the complex plane are the vertices of a parallelogram taken in order, if and only if

(a) $z_1 + z_4 = z_2 + z_3$ (b) $z_1 + z_3 = z_2 + z_4$

(c) $z_1 + z_2 = z_3 + z_4$ (d) None of these

Numerical Value Type Questions

53. For all complex numbers z of the form $1 + i\alpha, \alpha \in R$, if $z^2 = x + iy$, then value of $y^2 + 4x$ is

54. If $z = x - iy$ and $z^{1/3} = p + iq$, then $\frac{\left(\frac{x}{p}+\frac{y}{q}\right)}{\left(p^2+q^2\right)}$ equal to

55. If z_1, z_2 and z_3, z_4 are 2 pairs of complex conjugate numbers, and $\arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right)$ equals $k\pi$. Then value of k is

56. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, and $\arg z_1 - \arg z_2$ is equal to $k\pi$, then value of k is

57. If ω is a cube root of unity, then $(3 + 5\omega + 3\omega^2)^2 + (3 + 3\omega + 5\omega^2)^2$ is equal to

58. If z is any complex number such that $z+\frac{1}{z}=1$, then the value of $z^{99}+\frac{1}{z^{99}}$ is

59. If $\omega\,(\neq 1)$ is a cube root of unity and $(1+\omega)^7=A+B\omega$. Then, $A+2B$ equals to

60. The value of $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7}-i\cos\frac{2\pi k}{7}\right)$ is ki, then value of k is

61. If square root of $-7+24i$ is $x+iy$. If $x=\pm k$, then value of k is

62. If $|z+4|\leq 3$, then the maximum value of $|z+1|$ is

63. The number of complex numbers z such that $|z-1|=|z+1|=|z-i|$ equals

64. If z is a complex number such that $|z|\geq 2$, then the minimum value of $\left|z+\frac{1}{2}\right|$ is

65. The complex number z satisfies $z+|z|=2+8i$. The value of $|z|$ is

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

1. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is: **(2016)**

(a) $\frac{\pi}{6}$ (b) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

(c) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) $\frac{\pi}{3}$

2. Let $z = 1 + ai$ be a complex number, $a > 0$, such that z^3 is a real number. Then the sum $1 + z + z^2 + \ldots\ldots + z^{11}$ is equal to: **(2016/Online Set–2)**

(a) $-1250\sqrt{3}\,i$ (b) $1250\sqrt{3}\,i$

(c) $1365\sqrt{3}\,i$ (d) $-1365\sqrt{3}\,i$

3. Let $z \in C$, the set of complex numbers. Then the equation, $2|z + 3i| - |z - i| = 0$ represents : **(2017/Online Set–1)**

(a) a circle with radius $\frac{8}{3}$

(b) a circle with diameter $\frac{10}{3}$

(c) an ellipse with length of major axis $\frac{16}{3}$

(d) an ellipse with length of minor axis $\frac{16}{9}$

4. The equation $\text{Im}\left(\frac{iz-2}{z-i}\right)+1=0,\ z\in C,\ z\neq i$ represents a part of a circle having radius equal to : **(2017/Online Set–2)**

(a) 2 (b) 1

(c) $\frac{3}{4}$ (d) $\frac{1}{2}$

5. If $\alpha,\beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to : **(2018)**

(a) 2 (b) -1

(c) 0 (d) 1

6. The set of all $\alpha \in R$, for which $w = \frac{1+(1-8\alpha)z}{1-z}$ is a purely imaginary number, for all $z \in C$ satisfying $|z| = 1$ and $\text{Re}(z) \neq 1$, is : **(2018/Online Set–1)**

(a) an empty set (b) $\{0\}$

(c) $\left\{0, \frac{1}{4}, -\frac{1}{4}\right\}$ (d) equal to R

7. If $|z - 3 + 2i| \leq 4$ then the difference between the greatest value and the least value of $|z|$ is : **(2018/Online Set–2)**

(a) $2\sqrt{13}$ (b) 8

(c) $4+\sqrt{13}$ (d) $\sqrt{13}$

8. The least positive integer n for which $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^n = 1$, is: **(2018/Online Set–3)**

(a) 2 (b) 3

(c) 5 (d) 6

9. If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is : **(8-4-2019/Shift -1)**

(a) 2 (b) 5

(c) 4 (d) 3

10. If $z=\frac{\sqrt{3}}{2}+\frac{i}{2}\left(i=\sqrt{-1}\right)$, then $\left(1+iz+z^5+iz^8\right)^9$ is equal to : **(8-4-2019/Shift -2)**

(a) 0 (b) 1

(c) $\left(-1+2i\right)^9$ (d) -1

11. Let α and β be the roots of the equation $x^2+x+1=0$.

Then for $y\neq 0$ in R, $\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$ is equal to:

(9-4-2019/Shift -1)

(a) $y\left(y^2-1\right)$ (b) $y\left(y^2-3\right)$

(c) y^3 (d) y^3-1

12. Let z ∈ C be such that |z| < 1. If $\omega=\frac{5+3z}{5(1-z)}$, then:

(9-4-2019/Shift -2)

(a) $5\,\mathrm{Re}\,(\omega)>4$ (b) $4\,\mathrm{Im}\,(\omega)>5$

(c) $5\,\mathrm{Re}\,(\omega)>1$ (d) $5\,\mathrm{Im}\,(\omega)<1$

13. If a > 0 and $z=\frac{(1+i)^2}{a-i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to: **(10-4-2019/Shift -1)**

(a) $-\frac{1}{5}-\frac{3}{5}i$ (b) $-\frac{3}{5}-\frac{1}{5}i$

(c) $\frac{1}{5}-\frac{3}{5}i$ (d) $-\frac{1}{5}+\frac{3}{5}i$

14. If z and ω are two complex numbers such that $|z\omega|=1$ and $\arg(z)-\arg(\omega)=\frac{\pi}{2}$, then : **(10-4-2019/Shift -2)**

(a) $\bar{z}\omega=i$ (b) $z\bar{\omega}=\frac{-1+i}{\sqrt{2}}$

(c) $\bar{z}\omega=-i$ (d) $z\bar{\omega}=\frac{1-i}{\sqrt{2}}$

15. Let $z\in C$ *with* $\mathrm{Im}(z)=10$ and it satisfies $\frac{2z-n}{2z+n}=2i-1$ for some natural number n. Then : **(12-4-2019/Shift -2)**

(a) n = 20 and Re(z) = -10 (b) n = 40 and Re(z) = 10

(c) n = 40 and Re(z) = -10 (d) n = 20 and Re(z) -10

16. Let $A=\left\{\theta\in\left(-\frac{\pi}{2},\pi\right);\ \frac{3+2i\sin\theta}{1-2i\sin\theta}\text{ is purely imaginary}\right\}$

Then sum of the elements in A is: **(9-1-2019/Shift -1)**

(a) $\frac{5\pi}{6}$ (b) π

(c) $\frac{3\pi}{4}$ (d) $\frac{2\pi}{3}$

17. Let z_0 be a root of the quadratic equation, $x^2+x+1=0$. If $z=3+6iz_0^{81}-3iz_0^{93}$, then arg z is equal

(9-1-2019/Shift -2)

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$ (d) 0

18. Let $z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$. If R(z) and I(z) respectively denote the real and imaginary parts of z, then **(10-1-2019/Shift -1)**

(a) I(z) = 0 (b) R(z) > 0 and I(z) > 0

(c) R(z) < 0 and I(z) > 0 (d) R(z) = - 3

19. Let $\left(-2-\frac{1}{3}i\right)^3=\frac{x+iy}{27}\left(i=\sqrt{-1}\right)$, where x and y are real numbers then y- x equals: **(11-1-2019/Shift -1)**

20. Let z be a complex number such that |z| + z = 3 + i (where i = $\sqrt{-1}$). Then |z| is equal to :

(11-1-2019/Shift -2)

(a) $\frac{\sqrt{34}}{3}$ (b) $\frac{5}{3}$

(c) $\frac{\sqrt{41}}{4}$ (d) $\frac{5}{4}$

21. If $\frac{z-\alpha}{z+\alpha}(\alpha \in R)$ is a purely imaginary number and $|z|=2$, then a value of α is ______. **(12-1-2019/Shift -1)**

(a) 2 (b) 1

(c) $\frac{1}{2}$ (d) $\sqrt{2}$

22. Let z_1 and z_2 be two complex numbers satisfying $|z_1|=9$ and $|z_2-3-4i|=4$. Then the minimum value of $|z_1-z_2|$ is: **(12-1-2019/Shift -2)**

(a) 0 (b) $\sqrt{2}$

(c) 1 (d) 2

23. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is : **(2-9-2020/Shift -1)**

(a) $-\frac{1}{2}(1-i\sqrt{3})$ (b) $\frac{1}{2}(1-i\sqrt{3})$

(c) $-\frac{1}{2}(\sqrt{3}-i)$ (d) $\frac{1}{2}(\sqrt{3}-i)$

24. The imaginary part of $(3+2\sqrt{-54})^{1/2}-(3-2\sqrt{-54})^{1/2}$ can be : **(2-9-2020/Shift -2)**

(a) $\sqrt{6}$ (b) $-2\sqrt{6}$

(c) 6 (d) $-\sqrt{6}$

25. If $\left(\frac{1+i}{1-i}\right)^{\frac{m}{2}}=\left(\frac{1+i}{i-1}\right)^{\frac{n}{3}}=1, (m, n \in N)$ then the greatest common divisor of the least values of m and n is .... **(3-9-2020/Shift -1)**

26. If z_1, z_2 are complex numbers such that $\text{Re}(z_1)=|z_1-1|$, $\text{Re}(z_2)=|z_2-1|$ and arg $(z_1-z_2)=\frac{\pi}{6}$, then $\text{Im}(z_1+z_2)$ is equal to : **(3-9-2020/Shift -2)**

(a) $2\sqrt{3}$ (b) $\frac{2}{\sqrt{3}}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

27. Let $u=\frac{2z+i}{z-ki}, z=x+iy$ and k > 0. If the curve represented by Re(u) + Im (u) = 1 intersects the y-axis at the points P and Q where PQ = 5, then the value of k is : **(4-9-2020/Shift -1)**

(a) 4 (b) 1/2

(c) 2 (d) 3/2

28. If a and b are real numbers such that $(2+\alpha)^4=a+b\alpha$, where $\alpha=\frac{-1+i\sqrt{3}}{2}$ then a + b is equal to: **(4-9-2020/Shift -2)**

(a) 33 (b) 57

(c) 9 (d) 24

29. If the four complex numbers $z, \bar{z}, \bar{z}-2\text{Re}(\bar{z})$ and z-2Re(z) represent the vertices of a square of side 4 units in the Argand plane, the |z| is equal to: **(5-9-2020/Shift -1)**

(a) 2 (b) 4

(c) $4\sqrt{2}$ (d) $2\sqrt{2}$

30. The value of $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ is: **(5-9-2020/Shift -2)**

(a) $2^{15}i$ (b) -2^{15}

(c) $-2^{15}i$ (d) 6^5

31. The region represented by $\{z = x+iy \in C : |z| - \text{Re}(z) \le 1\}$ is also given by the inequality: (6-9-2020/Shift -1)

(a) $y^2 \le 2\left(x+\frac{1}{2}\right)$ (b) $y^2 \le x+\frac{1}{2}$

(c) $y^2 \ge 2\,(x+1)$ (d) $y^2 \ge x+1$

32. Let z = x + iy be a non-zero complex number such that $z^2 = i|z|^2$, where $i = \sqrt{-1}$, then z lies on the (6-9-2020/Shift -2)

(a) line, y = x (b) real axis

(c) imaginary axis (d) line, y = -x

33. If $\text{Re}\left(\frac{z-1}{2z+i}\right)=1$, where z = x + iy, then the point (x, y) lies on a (7-1-2020/Shift -1)

(a) circle whose centre is at $\left(-\frac{1}{2},-\frac{3}{2}\right)$

(b) straight line whose slope is $\left(\frac{3}{2}\right)$

(c) circle whose diameter is $\frac{\sqrt{5}}{2}$

(d) straight line whose slope is $\left(-\frac{2}{3}\right)$

34. If $\frac{3+i\sin\theta}{4-i\cos\theta}$, $\theta \in [0, 2\pi]$, is a real number, then the argument of $\sin\theta + i\cos\theta$ is (7-1-2020/Shift -2)

(a) $\pi - \tan^{-1}\left(\frac{4}{3}\right)$ (b) $-\tan^{-1}\left(\frac{3}{4}\right)$

(c) $\pi - \tan^{-1}\left(\frac{3}{4}\right)$ (d) $\tan^{-1}\left(\frac{4}{3}\right)$

35. If the equation, $x^2+bx+45=0\,(b \in R)$ has conjugate complex roots and they satisfy $|z+1| = 2\sqrt{10}$, then: (8-1-2020/Shift -1)

(a) $b^2+b=12$ (b) $b^2-b=42$

(c) $b^2-b=30$ (d) $b^2+b=72$

36. Let $\alpha = \frac{-1+i\sqrt{3}}{2}$. If $a=(1+\alpha)\sum_{k=0}^{100}\alpha^{2k}$ and $b=\sum_{k=0}^{100}\alpha^{3k}$, then a and b are the roots of the quadratic equation: (8-1-2020/Shift -2)

(a) $x^2+101x+100=0$ (b) $x^2+102x+101=0$

(c) $x^2-102x+101=0$ (d) $x^2-101x+100=0$

37. Let z be a complex number such that $\left|\frac{z-i}{z+2i}\right|=1$ and $|z|=\frac{5}{2}$. Then the value of $|z+3i|$ is (9-1-2020/Shift -1)

(a) $\sqrt{10}$ (b) $\frac{7}{2}$

(c) $\frac{15}{4}$ (d) $2\sqrt{3}$

38. If z be a complex number satisfying |Re(z)|+|Im(z)|=4, then |z| cannot be: (9-1-2020/Shift -2)

(a) $\sqrt{7}$ (b) $\sqrt{\frac{17}{2}}$

(c) $\sqrt{10}$ (d) $\sqrt{8}$

39. Let z be those comples numbers which satisfy $|z+5| \le 4$ and $z(1+i)+\bar{z}(1-i) \ge -10$, $i=\sqrt{-1}$. If the maximum value of $|z+1|^2$ is $\alpha+\beta\sqrt{2}$, then the value of $(\alpha+\beta)$ is ________. (26-02-2021/Shift-2)

40. The sum of 162^{th} power of the roots of the equation $x^3-2x^2+2x-1=0$ is (26-02-2021/Shift-1)

41. Let the lines $(2-i)z=(2+i)\bar{z}$ and $(2+i)z+(i-2)\bar{z}-4i=0$, (here $i^2=-1$) be normal to a circle C. If the line $iz+\bar{z}+1+i=0$ is tangent to this circle C, then its radius is: **(25-02-2021/Shift-1)**

(a) $\frac{3}{\sqrt{2}}$ (b) $\frac{3}{2\sqrt{2}}$

(c) $3\sqrt{2}$ (d) $\frac{1}{2\sqrt{2}}$

42. If $\alpha, \beta \in R$ are such that $1-2i$ (here $i^2=-1$) is a root $z^2+\alpha z+\beta=0$, then $(\alpha-\beta)$ is equal to: **(25-02-2021/Shift-2)**

(a) 3 (b) 7

(c) –7 (d) –3

43. Let $i=\sqrt{-1}$. If $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}}+\frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}}=k$, and $n=[|k|]$ be the greatest integral part of $|k|$. Then $\sum_{j=0}^{n+5}(j+5)^2-\sum_{j=0}^{n+5}(j+5)$ is equal to ______. **(24-02-2021/Shift-2)**

44. If the least and the largest real values of α, for which the equation $z+\alpha|z-1|+2i=0$ $(z \in C \text{ and } i=\sqrt{-1})$ has a solution, are p and q respectively; then $4(p^2+q^2)$ is equal to **(24-02-2021/Shift-1)**

45. If the equation $a|z|^2+\overline{\bar{\alpha}z+\alpha\bar{z}}+d=0$ represents a circle where a, d are real constants, then which of the following condition is correct? **(18-03-2021/Shift-1)**

(a) $|\alpha|^2-ad \neq 0$

(b) $|\alpha|^2-ad \geq 0$ and $a \in R$

(c) $\alpha=0$, $a, d \in R^+$

(d) $|\alpha|^2-ad>0$ and $a \in R-\{0\}$

46. Let z_1, z_2 be the roots of the equation $z^2+az+12=0$ and z_1, z_2 form an equilateral triangle with origin. Then, the value of $|a|$ is .... **(18-03-2021/Shift-1)**

47. Let a complex number be $w=1-\sqrt{3}i$. Let another complex number z be such that $|zw|=1$ and $\arg(z)-\arg(w)=\frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and ω is equal to : **(18-03-2021/Shift-2)**

(a) 4 (b) $\frac{1}{2}$

(c) 2 (d) $\frac{1}{4}$

48. If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x)=f(x^3)+x\,g(x^3)$ is divisible by x^2+x+1, then $P(1)$ is equal to ______. **(18-03-2021/Shift-2)**

49. Let a complex number $z, |z| \neq 1$, satisfy $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right) \leq 2$. Then, the largest value of $|z|$ is equal to ______. **(16-03-2021/Shift-1)**

(a) 7 (b) 6

(c) 5 (d) 8

50. Let z and ω be two complex numbers such that $\omega=z\bar{z}-2z+2$, $\left|\frac{z+i}{z-3i}\right|=1$ and $\operatorname{Re}(\omega)$ has minimum value. Then, the minimum value of $n \in N$ for which ω^n is real, is equal to ________. **(16-03-2021/Shift-1)**

51. The least value of |z| where z is complex number which satisfies the inequality $\exp\left(\frac{(|z|+3)(|z|-1)}{||z|+1|}\log_e 2\right) \geqslant \log_{\sqrt{2}}|5\sqrt{7}+9i|$ is equal to **(16-03-2021/Shift-2)**

(a) 8 (b) 3

(c) $\sqrt{5}$ (d) 2

52. The area of the triangle with vertices $A(z), B(iz)$ and $C(z+iz)$ is : **(17-03-2021/Shift-1)**

(a) $\frac{1}{2}|z+iz|^2$ (b) $\frac{1}{2}$

(c) $\frac{1}{2}|z|^2$ (d) 1

53. Let S_1, S_2 and S_3 be three sets defined as

$S_1 = \{z \in \mathbb{C} : |z-1| \le \sqrt{2}\}$

$S_2 = \{z \in \mathbb{C} : \text{Re}((1-i)z) \ge 1\}$

$S_3 = \{z \in \mathbb{C} : \text{Im}(z) \le 1\}$

Then the set $S_1 \cap S_2 \cap S_3$ **(17-03-2021/Shift-2)**

(a) is a singleton

(b) has exactly two elements

(c) has infinitely many elements

(d) has exactly three elements

54. If z and ω are two complex numbers such that $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \frac{3\pi}{2}$, then $\arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$ is:

(Here $\arg(z)$ denotes the principal argument of complex number z) **(20-07-2021/Shift-1)**

(a) $\frac{3\pi}{4}$ (b) $-\frac{\pi}{4}$

(c) $-\frac{3\pi}{4}$ (d) $\frac{\pi}{4}$

55. The point P (a, b) undergoes the following three transformations successively:

(1) reflection about the line y = x.

(2) translation through 2 units along the positive direction of x-axis.

(3) rotation through angle $\frac{\pi}{4}$ about the origin in the anti-clockwise direction.

If the co-ordinates of the final position of the point P are $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$, then the value of $2a+b$ is equal to **(27-07-2021/Shift-2)**

(a) 9 (b) 5

(c) 13 (d) 7

56. Let C be the set of all complex numbers. Let $S_1 = \{z \in C : |z-2| \le 1\}$ and

$S_2 = \{z \in C : z(1+i) + \bar{z}(1-i) \ge 4\}$ Then, the maximum value of $\left|z - \frac{5}{2}\right|^2$ for $z \in S_1 \cap S_2$ is equal to :

(27-07-2021/Shift-2)

(a) $\frac{3+2\sqrt{2}}{4}$ (b) $\frac{5+2\sqrt{2}}{2}$

(c) $\frac{3+2\sqrt{2}}{2}$ (d) $\frac{5+2\sqrt{2}}{4}$

57. If the real part of the complex number

$z = \frac{3+2i\cos\theta}{1-3i\cos\theta}, \theta \in \left(0, \frac{\pi}{2}\right)$ is zero, then the value of $\sin^2 3\theta + \cos^2\theta$ is equal to ____________.

(27-07-2021/Shift-2)

58. Let n denote the number of solutions of the equation $z^2 + 3\bar{z} = 0$, z is a comples number. Then the value of $\sum_{k=0}^{\infty} \frac{1}{n^k}$ is equal to: **(22-07-2021/Shift-2)**

(a) 1 (b) 2

(c) $\frac{4}{3}$ (d) $\frac{3}{2}$

59. The equation of a circle is

$\text{Re}(z^2) + 2(\text{Im}(z))^2 + 2\text{Re}(z) = 0$, where z = x + iy. A line which passes through the center of the given circle and the vertex of the parabola, $x^2 - 6x - y + 13 = 0$ has y-intercept equal to **(25-07-2021/Shift-2)**

60. If for the complex numbers z satisfying $|z-2-2i| \le 1$, the maximum value of $|3iz+6|$ is attained at $a+ib$, then $a+b$ is equal to __________ ? **(01-09-2021/Shift-2)**

61. The least positive integer n such that $\frac{(2i)^n}{(1-i)^{n-2}}, i=\sqrt{-1}$, is a positive integer is **(26-08-2021/Shift-2)**

62. If $S=\left\{z\in C:\frac{z-i}{z+2i}\in R\right\}$, then: **(27-08-2021/Shift-1)**

(a) S is a circle in the complex plane
(b) S contains only one element
(c) S is a straight line in the complex plane
(d) S contains exactly two elements

63. The equation $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$ represents a circle with : **(26-08-2021/Shift-1)**

(a) centre at $(0,-1)$ and radius $\sqrt{2}$

(b) centre at $(0,1)$ and radius 2

(c) centre at $(0,1)$ and radius $\sqrt{2}$

(d) centre at $(0,0)$ and radius $\sqrt{2}$

64. Let $z=\frac{1-i\sqrt{3}}{2}, i=\sqrt{-1}$. Then the value of

$$21+\left(z+\frac{1}{z}\right)^3+\left(z^2+\frac{1}{z^2}\right)^3+\left(z^3+\frac{1}{z^3}\right)^3+ \ldots\ldots+\left(z^{21}+\frac{1}{z^{21}}\right)^3$$ is _____.

(26-08-2021/Shift-1)

65. Let z_1 and z_2 $\arg(z_1-z_2)=\frac{\pi}{4}$ and z_1, z_2 satisfy the equation $|z-3|=\mathrm{Re}(z)$. Then the imaginary part of z_1+z_2 is equal to _______. **(27-08-2021/Shift-2)**

66. A point z moves in the complex plane such that $\arg\left(\frac{z-2}{z+2}\right)=\frac{\pi}{4}$, then the minimum value of $\left|z-9\sqrt{2}-2i\right|^2$ is equal to ___________ ?

(31-08-2021/Shift-1)

67. If $a_r=\cos\frac{2r\pi}{9}+i\sin\frac{2r\pi}{9}, r=1,2,3,\ldots, i=\sqrt{-1}$, then the determinant $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is equal to?

(31-08-2021/Shift-1)

(a) $a_2a_6-a_4a_8$ (b) $a_1a_9-a_3a_7$

(c) a_5 (d) a_9

68. If z is a complex number such that $\frac{z-i}{z-1}$ is purely imaginary, then the minimum value of $|z-(3+3i)|$ is

(31-08-2021/Shift-2)

(a) $3\sqrt{2}$ (b) $6\sqrt{2}$

(c) $2\sqrt{2}$ (d) $2\sqrt{2}-1$

69. If $\left(\sqrt{3}+i\right)^{100}=2^{99}(p+iq)$, then p and q are roots of the equation: **(26-08-2021/Shift-2)**

(a) $x^2+\left(\sqrt{3}+1\right)x+\sqrt{3}=0$

(b) $x^2+\left(\sqrt{3}-1\right)x-\sqrt{3}=0$

(c) $x^2-\left(\sqrt{3}+1\right)x+\sqrt{3}=0$

(d) $x^2-\left(\sqrt{3}-1\right)x-\sqrt{3}=0$

70. Let C be the set of all complex numbers. Let

$S_1=\{z\in C \,|\, |z-3-2i|^2=8\}$

$S_2=\{z\in C \,|\, \mathrm{Re}(z)\geq 5\}$ and

$S_3=\{z\in C \,|\, |z-\bar{z}|\geq 8\}$.

Then the number of elements in $S_1\cap S_2\cap S_3$ is equal to:

(27-07-2021/Shift-1)

(a) 1 (b) 0

(c) Infinite (d) 2

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

Objective Questions I [Only one correct option]

1. $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ will be purely imaginary, if θ is equal to

(a) $2n\pi\pm\frac{\pi}{3}$ (b) $n\pi+\frac{\pi}{3}$

(c) $n\pi\pm\frac{\pi}{3}$ (d) None of these

2. If (a + ib) (c + id) (e + if) (g + ih) = A + i B, then $(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2)$ is equal to

(a) A^2-B^2 (b) A^2+B^2

(c) A^4+B^4 (d) A^4-B^4

3. The area of the triangle on the complex plane formed by the complex numbers z, iz and z + iz is

(a) $|z|^2$ (b) $|\bar{z}|^2$

(c) $\frac{|z|^2}{2}$ (d) none of these

4. If $\arg(\bar{z}_1)=\arg(z_2)$, then

(a) $z_2=kz_1^{-1}(k>0)$ (b) $z_2=kz_1(k>0)$

(c) $|z_2|=|\bar{z}_1|$ (d) None of these

5. z and ω are two non zero complex number such that $|z|=|\omega|$ and $\operatorname{Arg} z+\operatorname{Arg}\omega=\pi$, then z equals

(a) $\bar{\omega}$ (b) $-\bar{\omega}$

(c) ω (d) $-\omega$

6. If z is a complex number of unit modulus and argument θ, then $\arg\left(\frac{1+z}{1+\bar{z}}\right)$ is equal to

(a) $-\theta$ (b) $\frac{\pi}{2}-\theta$

(c) θ (d) $\pi-\theta$

7. If $\theta\in(0,\pi)$, the principal value of the arg (z) and |z| of the complex number $z=\frac{(1+\cos\theta+i\sin\theta)^5}{(\cos\theta+i\sin\theta)^3}$ is

(a) $-\frac{\theta}{2},32\cos^5\frac{\theta}{2}$ (b) $\frac{\theta}{2},32\cos^5\frac{\theta}{2}$

(c) $-\frac{\theta}{2},16\cos^4\frac{\theta}{2}$ (d) None of these

8. If $z=re^{i\theta}$, then $|i^{iz}|$ is equal to

(a) $e^{-r\sin\theta}$ (b) $re^{-r\sin\theta}$

(c) $e^{-\frac{\pi}{2}r\cos\theta}$ (d) $re^{-r\cos\theta}$

9. The minimum value of $|Z-1+2i|+|4i-3-Z|$ is

(a) $\sqrt{5}$ (b) 5

(c) $2\sqrt{13}$ (d) $\sqrt{15}$

10. If $|z^2-1|=|z|^2+1$, then z lies on

(a) a circle (b) the imaginary axis

(c) the real axis (d) an ellipse

11. If $\left|z-\frac{4}{z}\right|=2$, then the maximum value of $|z|$ is equal to

(a) $\sqrt{3}+1$ (b) $\sqrt{5}+1$

(c) 2 (d) $2+\sqrt{2}$

12. If $z^2-z+1=0$, then z^n-z^{-n}, where n is a multiple of 3, is

(a) $2(-1)^n$ (b) 0

(c) $(-1)^{n+1}$ (d) None of these

13. The value of the expression

$$2\left(1+\frac{1}{\omega}\right)\left(1+\frac{1}{\omega^2}\right)+3\left(2+\frac{1}{\omega}\right)\left(2+\frac{1}{\omega^2}\right)$$

$$+4\left(3+\frac{1}{\omega}\right)\left(3+\frac{1}{\omega^2}\right)+\ldots\ldots+(n+1)\left(n+\frac{1}{\omega}\right)\left(n+\frac{1}{\omega^2}\right),$$

where ω is an imaginary cube root of unity, is

(a) $\frac{n(n^2+2)}{3}$ (b) $\frac{n(n^2-2)}{3}$

(c) $\frac{n^2(n+1)^2+4n}{4}$ (d) None of these

14. Which of the following is a fourth root of $\frac{1}{2}+i\frac{\sqrt{3}}{2}$?

(a) $\text{cis}\frac{\pi}{12}$ (b) $\text{cis}\frac{\pi}{2}$

(c) $\text{cis}\frac{\pi}{6}$ (d) $\text{cis}\frac{\pi}{3}$

15. If α, β, γ are the cube roots of a negative number p, then for any three real numbers x, y, z the value of

$\frac{x\alpha+y\beta+z\gamma}{x\beta+y\gamma+z\alpha}$ is

(a) $\frac{1-i\sqrt{3}}{2}$ (b) $\frac{-1-i\sqrt{3}}{2}$

(c) $(x+y+z)\,i$ (d) π

16. The complex number $z = 1 + i$ is rotated through an angle $\frac{3\pi}{2}$ in anticlockwise direction about the origin and stretched by additional $\sqrt{2}$ units, then the new complex number is

(a) $-\sqrt{2}-\sqrt{2}i$ (b) $\sqrt{2}-\sqrt{2}i$

(c) $2-\sqrt{2}i$ (d) $2-2i$

17. The equation $|z+1-i| = |z-1+i|$ represents a

(a) straight line (b) circle

(c) parabola (d) hyperbola

18. If z_1 and z_2 are two complex numbers satisfying the equation $\left|\frac{z_1+iz_2}{z_1-iz_2}\right|=1$, then $\frac{z_1}{z_2}$ is

(a) purely real (b) of unit modulus

(c) purely imaginary (d) None of these

19. If $\left|\frac{z_1z-z_2}{z_1z+z_2}\right|=K$, $K>0$ $(z_1, z_2 \neq 0)$, then

(a) for $k \neq 1$, locus z is a straight line

(b) for $k \notin \{1, 0\}$, z lies on a circle

(c) for $k \neq 0$, z represents a point

(d) for $k \neq 1$, z lies on the perpendicular bisector of the line segment joining $\frac{z_2}{z_1}$ and $-\frac{z_2}{z_1}$.

20. If the complex numbers z_1, z_2, z_3 satisfying $\frac{z_1-z_3}{z_2-z_3}=\frac{1-i\sqrt{3}}{2}$, then triangle is

(a) an equilateral triangle

(b) a right angled triangle

(c) a acute angled triangle

(d) an obtuse angled isosceles triangle

21. If A and B be two complex numbers satisfying $\frac{A}{B}+\frac{B}{A}=1$. Then the two points represented by A and B and the origin form the vertices of

(a) an equilateral triangle

(b) an isosceles triangle which is not equilateral

(c) an isosceles triangle which is not right angled

(d) a right angled triangle

22. Let z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, z being complex. Further, assume that the origin, z_1 and z_2 form an equilateral triangle, then

(a) $a^2 = 2b$ (b) $a^2 = 3b$

(c) $c^2 = ab$ (d) $a^2 = b$

23. Let z_1, z_2 and z_3 be three points on $|z| = 1$. If θ_1, θ_2 and θ_3 be the arguments of z_1, z_2, z_3 respectively, then $\cos(\theta_1-\theta_2)+\cos(\theta_2-\theta_3)+\cos(\theta_3-\theta_1)$

(a) $\geq -\frac{3}{2}$ (b) $\leq -\frac{3}{2}$

(c) $\geq \frac{3}{2}$ (d) none of these

24. If $z_n = \cos\frac{\pi}{n(n+1)(n+2)} + i\sin\frac{\pi}{n(n+1)(n+2)}$ for n=1, 2, 3, ..., k, then the value of $\lim_{k\to\infty}(z_1 z_2 ..... z_k)$ is

(a) $\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$ (b) $-\frac{1}{2} + i\frac{\sqrt{3}}{2}$

(c) $-\frac{1}{2} - i\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$

25. If z is a complex number satisfying $|z^2 - 1| = 4|z|$, then the minimum value of |z| is

(a) $2\sqrt{5}+4$ (b) $2\sqrt{5}-4$

(c) $\sqrt{5}-2$ (d) None of these

26. The area of the triangle whose vertices are i, α, β, where $i=\sqrt{-1}$ and α, β are the non-real cube roots of unity, is

(a) $\frac{3\sqrt{3}}{2}$ (b) $\frac{3\sqrt{3}}{4}$

(c) 0 (d) $\frac{\sqrt{3}}{4}$

27. The roots of $(z-1)^n = 2\omega(z+1)^n$ (where n ≥ 3 and ω is complex cube root of unity) lie on a

(a) straight line (b) ellipse

(c) circle (d) rectangular hyperbola

28. Let P denotes a complex number z on the Argand's plane, and Q denotes a complex number $\sqrt{2|z|^2}\, \text{CiS}\left(\frac{\pi}{4}+\theta\right)$ where θ = amp z. If 'O' is the origin, then the ΔOPQ is

(a) isosceles but not right angled

(b) right angled but not isosceles

(c) right isosceles

(d) equilateral

29. If z = x + iy and $w = \frac{1-iz}{z-i}$, then |w| = 1 implies that in the complex plane

(a) z lies on the imaginary axis

(b) z lies on the real axis

(c) z lies on the unit circle

(d) None of the above

30. If z_1 lies in |z–3| < 4, z_2 on |z–1| + |z +1| = 3 and $A = |z_1 - z_2|$, then

(a) $0 \le A \le \frac{15}{2}$ (b) $0 < A \le \frac{15}{2}$

(c) $0 \le A \le \frac{17}{2}$ (d) $0 \le A < \frac{17}{2}$

31. The system of equations $\left.\begin{array}{l}|z+1+i| = \sqrt{2}\\ |z| = 3\end{array}\right\}$, (where $i=\sqrt{-1}$) has

(a) no solution (b) one solution

(c) two solutions (d) none of these

32. If $|z| = \max\{|z-1|, |z+1|\}$ then

(a) $|z+\bar{z}| = \frac{1}{2}$ (b) $z - \bar{z} = 1$

(c) $|z+\bar{z}| = 1$ (d) None of these

33. Locus of z, if

$$\arg[z-(1+i)] = \begin{cases} \frac{3\pi}{4}, & \text{when } |z| \le |z-2| \\ \frac{-\pi}{4}, & \text{when } |z| > |z-2| \end{cases}$$ is

(a) straight line passing through (2, 0)

(b) straight lines passing through (2, 0), (1, 1)

(c) a line segment

(d) a set of two rays

Objective Questions II [One or more than one correct option]

34. If z_1 = a + ib and z_2 = c + id are complex numbers such that $|z_1| = |z_2| = 1$ and Re $(z_1\bar{z}_2) = 0$, then the pair of complex numbers w_1 = a + ic and w_2 = b + id satisfies

(a) $|w_1| = 1$ (b) $|w_2| = 1$

(c) $\text{Re}(w_1\bar{w}_2) = 0$ (d) None of the above

35. Let z_1 and z_2 be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\frac{z_1+z_2}{z_1-z_2}$ may be

(a) zero (b) real and positive

(c) real and negative (d) purely imaginary

36. Let z_1, z_2 be two complex numbers represented by points on the circle $|z| = 1$ and $|z| = 2$, respectively, then

(a) max $|2z_1+z_2| = 4$ (b) min $|z_1-z_2| = 1$

(c) $\left|z_2+\frac{1}{z_1}\right|\leq 3$ (d) None of these

37. If a complex number z has modulus 1 and argument $\pi/3$, then $z^2 + z$

(a) is purely imaginary

(b) has modulus $\sqrt{3}$

(c) lies on the imaginary axis

(d) none of these

38. If α is an imaginary constant such that $\alpha z^2 + z + \bar{\alpha} = 0$ has a real root, then

(a) $\alpha+\bar{\alpha}=1$

(b) $\alpha+\bar{\alpha}=0$

(c) $\alpha+\bar{\alpha}=-1$

(d) the absolute value of the real root is 1

39. If $\omega \neq 1$ is a complex cube root of unity, then sum of the series $S = 1 + 2\omega + 3\omega^2 + ... + 3n\omega^{3n-1}$ $(n \in N)$ is

(a) $\frac{3n}{\omega-1}$ (b) $n\left(\omega^2-1\right)$

(c) 0 (d) 1

40. If z_0, z_1 represents points P, Q on the locus $|z-1|=1$ and the line segment PQ subtends an angle $\frac{\pi}{2}$ at the points $z = 1$, then z_1 is equal to

(a) $1 + i(z_0-1)$ (b) $\frac{i}{z_0-1}$

(c) $1 - i(z_0-1)$ (d) $i(z_0-1)$

41. If A (z_1), B (z_2) and C (z_3) be the vertices of a triangle ABC in which $\angle ABC = \frac{\pi}{4}$ and $\frac{AB}{BC} = \sqrt{2}$, then the value of z_2 is equal to

(a) $z_3 + i(z_1 + z_3)$ (b) $z_3 - i(z_1 - z_3)$

(c) $z_3 + i(z_1 - z_3)$ (d) None of these

42. Suppose $A(z_1)$, $B(z_2)$ and $C(z_3)$ are vertices of a triangle lying on the unit circle $|z| = 1$. AD is altitude of the ΔABC meeting the unit circle in E.

(a) orthocentre of ΔABC is $z_1 + z_2 + z_3$

(b) affix of E is $-z_2z_3/z_1$

(c) if $z_1^2 = z_2z_3$ and $z_2^2 = z_3z_1$, then ΔABC is equilateral

(d) if $z_2 + z_3 = 0$, then ΔABC is a right angled.

43. If z_1, z_2, z_3, z_4 are roots of the equation

$$a_0z^4 + a_1z^3 + a_2z^2 + a_3z + a_4 = 0,$$

where a_0, a_1, a_2, a_3 and a_4 are real, then

(a) $\bar{z}_1, \bar{z}_2, \bar{z}_3, \bar{z}_4$ are also roots of the equation

(b) z_1 is equal to at least one of $\bar{z}_1, \bar{z}_2, \bar{z}_3, \bar{z}_4$

(c) $-\bar{z}_1, -\bar{z}_2, -\bar{z}_3, -\bar{z}_4$ are also roots of the equation

(d) None of the above

Numerical Value Type Questions

44. The value of $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^6 + \left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\right)^6$ is

45. If $z^2 + z + 1 = 0$, where z is a complex number, then the value of $\left(z+\frac{1}{z}\right)^2 + \left(z^2+\frac{1}{z^2}\right)^2 + \left(z^3+\frac{1}{z^3}\right)^2 + ... + \left(z^6+\frac{1}{z^6}\right)^2$ is

46. Number of common roots of the equations $z^3 + 2z^2 + 2z + 1 = 0$ and $z^{1985} + z^{100} + 1 = 0$, z being a complex number, is

47. If α and β are different complex numbers with $|\beta| = 1$, then $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$ is

48. For a complex number z the minimum value of $|z| + |z-2|$ is

49. If a,b,c are three distinct real numbers and $\omega \neq 1$ is a complex cube root of unity, then the value of $\left|\dfrac{a+b\omega+c\omega^2}{a\omega^2+b+c\omega}\right|$ is

50. If z lies on the circle centered at origin and if area of the triangle, whose vertices are z, ωz and $z+\omega z$, (ω being an imaginary cube root of unity), is $4\sqrt{3}$ sq. units. Then radius of the circle is

51. If 1, x_1, x_2, x_3 are the roots of $x^4 - 1 = 0$ and ω is an imaginary cube root of unity, then the value of

$$\frac{(\omega^2-x_1)(\omega^2-x_2)(\omega^2-x_3)}{(\omega-x_1)(\omega-x_2)(\omega-x_3)}$$ is

52. If $z \in C$, then $\left|\dfrac{2z-i}{5z+1}\right| = m$, $m \in R$ represents a straight line if 10 m =

53. If a point z_1 is the reflection of a point z_2 through the line $b\bar{z}+\bar{b}z = c$, $b \neq 0$, in the Argand plane, then $\bar{b}z_2 + b\bar{z}_1$ is equal to kc, then value of k is

54. Number of complex numbers z such that $|z| = 1$ and $\left|\dfrac{z}{\bar{z}}+\dfrac{\bar{z}}{z}\right| = 1$ is

Assertion & Reason

(A) **If ASSERTION is true, REASON is true, REASON is a correct explanation for ASSERTION.**

(B) **If ASSERTION is true, REASON is true, REASON is not a correct explanation for ASSERTION.**

(C) **If ASSERTION is true, REASON is false.**

(D) **If ASSERTION is false, REASON is true.**

55. **Assertion :** If z is a complex number ($z \neq 1$), then $\left|\dfrac{z}{|z|}-1\right| \leq |\arg z|$

Reason : In a unit radius circle chord (AP) $\leq$ arc (AP)

(a) A (b) B
(c) C (d) D

56. **Assertion :** If $|z| \geq 2$, then the least value of $\left|z+\dfrac{1}{2}\right|$ is $\dfrac{3}{2}$.

Reason : $|z_1+z_2| \leq |z_1|+|z_2|$

(a) A (b) B
(c) C (d) D

Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching. For each question, choose the option corresponding to the correct matching.

57. Match the following

	Column I		Column II
(A)	Locus of the point z satisfying the equation $\text{Re}(z^2) = \text{Re}(z+\bar{z})$	**(P)**	A circle
(B)	Locus of the point z satisfying the equation $|z-z_1|+|z-z_2| = \lambda$, $\lambda \in R^+$ and $\lambda \not< |z_1-z_2|$	**(Q)**	A straight line
(C)	Locus of the point z satisfying the equation $\left|\dfrac{2z-i}{z+1}\right| = m$, where $i=\sqrt{-1}$, $m \in R^+$	**(R)**	An ellipse
(D)	If $|\bar{z}| = 25$, then the points representing the complex number $-1+75\bar{z}$ will be on	**(S)**	A rectangular hyperbola

The correct matching is

(a) A–S; B–Q,R; C–Q,P; D–P
(b) A–Q; B–P; C–Q,P; D–S
(c) A–P; B–Q; C–R; D–S
(d) A–S; B–Q; C–R; D–P

58. Match the equation on the left with the curve they represent on the right

Column 1	Column 2
(A) $|z-3|+|z-i| = 10$	**(P)** circle
(B) $\left|\dfrac{2z-3}{z-i}\right| = 2$	**(Q)** hyperbola
(C) $z^2+\bar{z}^2 = 5$	**(R)** straight line
(D) $\left|\dfrac{z-6}{z-2i}\right| = 3$	**(S)** ellipse

The correct matching is

(a) A-S; B-R; C-P; D-Q
(b) A-S; B-P; C-Q; D-P
(c) A-S; B-R; C-R; D-P
(d) A-S; B-R; C-Q; D-P

Using the following passage, solve Q.59 to Q.62

Passage – 1

Let $w = \cos\left(\frac{2\pi}{7}\right) + i\sin\left(\frac{2\pi}{7}\right)$ and $\alpha = w + w^2 + w^4$ and $\beta = w^3 + w^5 + w^6$.

59. $\alpha\beta$ equals :

(a) -1 (b) 0

(c) 1 (d) 2

60. α and β are roots of the equations :

(a) $x^2+x+1=0$ (b) $x^2+x+2=0$

(c) $x^2+3x+5=0$ (d) None of these

61. 2α equals :

(a) $-1+\sqrt{7}i$ (b) $-1-\sqrt{7}i$

(c) $1+7i$ (d) $1-7i$

62. $\sum_{k=0}^{6} w^{k^2}$ equals :

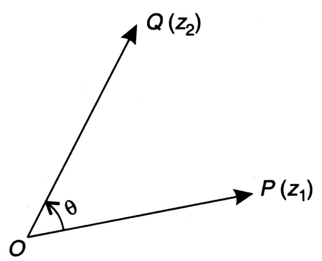
(a) i (b) $\sqrt{7}i$

(c) $-i$ (d) $-\sqrt{7}i$

Using the following passage, solve Q.63 to Q.65

Passage – 2

Let $z_1 = a_1 + ib_1 \equiv (a_1, b_1)$ and $z_2 = a_2 + ib_2 \equiv (a_2, b_2)$; where $i = \sqrt{-1}$, be two complex numbers.



If $\angle POQ = \theta$, From Rotation theorem

$$\frac{z_2 - 0}{z_1 - 0} = \frac{|z_2|}{|z_1|}e^{i\theta} \Rightarrow \frac{z_2\bar{z}_1}{z_1\bar{z}_1} = \frac{|z_2|}{|z_1|}e^{i\theta}$$

$$\Rightarrow \frac{z_2\bar{z}_1}{|z_1|^2} = \frac{|z_2|}{|z_1|}e^{i\theta} \Rightarrow z_2\bar{z}_1 = |z_1||z_2|e^{i\theta}$$

$$\Rightarrow z_2\bar{z}_1 = |z_1||z_2|(\cos\theta + i\sin\theta)$$

$$\therefore \text{Re}(z_2\bar{z}_1) = |z_1||z_2|\cos\theta \quad \text{...(i)}$$

and $\text{Im}(z_2\bar{z}_1) = |z_1||z_2|\sin\theta \quad \text{...(ii)}$

The dot product of z_1 and z_2 is defined by $z_1 . z_2 = |z_1| |z_2| \cos\theta = \text{Re}(z_2\bar{z}_1)$ [from (i)] and cross product of z_1 and z_2 is defined $z_1 \times z_2 = |z_1||z_2| \sin\theta = \text{Im}(z_2\bar{z}_1)$ [from Eq. (ii)]

63. If $z_1 = 2+5i$, $z_2 = 3-i$, then the value of $\sqrt{(z_1.z_2 + z_2 \times z_1)}$ is equal to

(a) 2 (b) 3

(c) $2\sqrt{3}$ (d) $3\sqrt{2}$

64. If $z_1 = 3 + 4i$ and $z_2 = 4 + 3i$, then the value of $\sin\theta\left(\pi < \theta < \frac{3\pi}{2}\right)$ is equal to

(a) $-\frac{1}{7}$ (b) $-\frac{7}{25}$

(c) $-\frac{24}{25}$ (d) $-\frac{1}{25}$

65. If $z_1 = 5 + 12i$ and $z_2 = 3 + 4i$, then (the projection of z_1 on z_2 + projection of z_2 on z_1) is equal to

(a) $\frac{4131}{65}$ (b) $\frac{3411}{65}$

(c) $\frac{1134}{65}$ (d) $\frac{1341}{65}$

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Objective Questions I [Only one correct option]

1. Let z_1 and z_2 be n^{th} roots of unity which subtend a right angle at the origin, then n must be of the form (where k is an integer) **(2001)**

(a) $4k+1$ (b) $4k+2$

(c) $4k+3$ (d) $4k$

2. The complex numbers z_1, z_2 and z_3 satisfying $\frac{z_1-z_3}{z_2-z_3}=\frac{1-i\sqrt{3}}{2}$ are the vertices of a triangle which is **(2001)**

(a) of area zero (b) right-angled isosceles

(c) equilateral (d) obtuse-angled isosceles

3. Let $\omega=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$, then value of the determinant

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$$ is **(2002)**

(a) 3ω (b) $3\omega(\omega-1)$

(c) $3\omega^2$ (d) $3\omega(1-\omega)$

4. For all complex numbers z_1, z_2 satisfying $|z_1|=12$ and $|z_2-3-4i|=5$, the minimum value of $|z_1-z_2|$ is **(2002)**

(a) 0 (b) 2

(c) 7 (d) 17

5. If $|z|=1$ and $w=\frac{z-1}{z+1}(\text{where } z\neq -1)$, then Re (w) is **(2003)**

(a) 0 (b) $\frac{1}{|z+1|^2}$

(c) $\left|\frac{1}{z+1}\right|.\frac{1}{|z+1|^2}$ (d) $\frac{\sqrt{2}}{|z+1|^2}$

6. If $\omega(\neq 1)$ be a cube root of unity and $(1+\omega^2)^n=(1+\omega^4)^n$, then the least positive value of n is **(2004)**

(a) 2 (b) 3

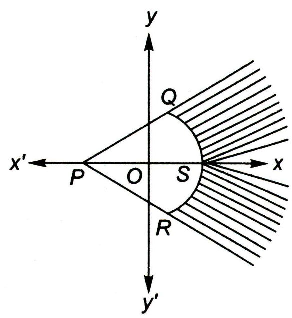
(c) 5 (d) 6

7. The minimum value of $|a+b\omega+c\omega^2|$, where a, b and c are all not equal integers and $\omega(\neq 1)$ is a cube root of unity, is **(2005)**

(a) $\sqrt{3}$ (b) $\frac{1}{2}$

(c) 1 (d) 0

8. The shaded region, where $P=(-1,0)$, $Q=(-1+\sqrt{2},\sqrt{2})$, $R=(-1+\sqrt{2},-\sqrt{2})$, $S=(1,0)$ is represented by **(2005)**



(a) $|z+1|>2, |\arg(z+1)|<\frac{\pi}{4}$

(b) $|z+1|<2, |\arg(z+1)|<\frac{\pi}{2}$

(c) $|z+1|>2, |\arg(z+1)|>\frac{\pi}{4}$

(d) $|z-1|<2, |\arg(z+1)|>\frac{\pi}{2}$

9. If $w = \alpha + i\beta$, where $\beta \neq 0$ and $z \neq 1$, satisfies the condition that $\left(\frac{w - \bar{w}z}{1 - z}\right)$ is purely real, then the set of values of z is **(2006)**

(a) $|z| = 1,\ z \neq 2$ (b) $|z| = 1$ and $z \neq 1$

(c) $z = \bar{z}$ (d) None of the above

10. A man walks a distance of 3 units from the origin towards the north-east (N 45° E) direction. From there, he walks a distance of 4 units towards the north-west (N 45°W) direction to reach a point P. Then, the position of P in the Argand plane is **(2007)**

(a) $3e^{i\pi/4} + 4i$ (b) $(3 - 4i)e^{i\pi/4}$

(c) $(4 - 3i)e^{i\pi/4}$ (d) $(3 + 4i)e^{i\pi/4}$

11. If $|z| = 1$ and $z \neq \pm 1$, then all the values of $\frac{z}{1 - z^2}$ lie on **(2007)**

(a) a line not passing through the origin

(b) $|z| = \sqrt{2}$

(c) the x-axis

(d) the y-axis

12. A particle P starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{-1}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1. From z_1 the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i} + \hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by **(2008)**

(a) $6 + 7i$ (b) $-7 + 6i$

(c) $7 + 6i$ (d) $-6 + 7i$

13. Let $z = \cos\theta + i\sin\theta$. Then, the value of $\sum_{m=1}^{15} \text{Im}\,(z^{2m-1})$ at $\theta = 2°$ is **(2009)**

(a) $\frac{1}{\sin 2°}$ (b) $\frac{1}{3\sin 2°}$

(c) $\frac{1}{2\sin 2°}$ (d) $\frac{1}{4\sin 2°}$

14. Let $z = x + iy$ be a complex number where x and y are integers. Then, the area of the rectangle whose vertices are the roots of the equation $z\bar{z}^3 + \bar{z}z^3 = 350$ is **(2009)**

(a) 48 (b) 32

(c) 40 (d) 80

15. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$, where each of a, b and c is either ω or ω^2. Then the number of distinct matrices in the set S is **(2011)**

(a) 2 (b) 6

(c) 4 (d) 8

16. Let z be a complex number such that the imaginary part of z is non-zero and $a = z^2 + z + 1$ is real. Then, a cannot take the value **(2012)**

(a) -1 (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

17. Let complex numbers α and $\frac{1}{\bar{\alpha}}$ lies on circles

$(x - x_0)^2 + (y - y_0)^2 = r^2$ and

$(x - x_0)^2 + (y - y_0)^2 = 4r^2$, respectively.

If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$, then $|\alpha|$ is equal to **(2013)**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{7}}$ (d) $\frac{1}{3}$

18. Let S be the set of all complex numbers z satisfying $|z-2+i|\geq\sqrt{5}$. If the complex number z_0 is such that $\frac{1}{|z_0-1|}$ is the maximum of the set $\left\{\frac{1}{|z-1|}: z\in S\right\}$, then the principal argument of $\frac{4-z_0-\bar{z}_0}{z_0-\bar{z}_0+2i}$ is **(2019)**

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$

(c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

19. Let $\theta_1,\theta_2,....\theta_{10}$ be positive valued angles (in radian) such that $\theta_1+\theta_2+....+\theta_{10}=2\pi$. Define the complex numbers $z_1=e^{i\theta_1}, z_k=z_{k-1}e^{i\theta_k}$ for $k=2,3,...,10$ where $i=\sqrt{-1}$. Consider the statements P and Q given below:

$$P:|z_2-z_1|+|z_3-z_2|+...+|z_{10}-z_9|+|z_1-z_{10}|\leq 2\pi$$

$$Q:|z_2^2-z_1^2|+|z_3^2-z_2^2|+....+|z_{10}^2-z_9^2|+|z_1^2-z_{10}^2|\leq 4\pi$$

Then, **(2021)**

(a) P is TRUE and Q is FALSE

(b) Q is TRUE and P is FALSE

(c) Both P and Q are TRUE

(d) Both P and Q are FALSE

Objective Questions II [One or more than one correct option]

20. Let z_1 and z_2 be two distinct complex numbers and let $z=(1-t)z_1+tz_2$ for some real number t with $0<t<1$. If arg (w) denotes the principal argument of a non-zero complex number w, then **(2010)**

(a) $|z-z_1|+|z-z_2|=|z_1-z_2|$

(b) $\arg(z-z_1)=\arg(z-z_2)$

(c) $\begin{vmatrix} z-z_1 & \bar{z}-\bar{z}_1 \\ z_2-z_1 & \bar{z}_2-\bar{z}_1 \end{vmatrix}=0$

(d) $\arg(z-z_1)=\arg(z_2-z_1)$

21. Let $w=\frac{\sqrt{3}+i}{2}$ and $P=\{w^n: n=1,2,3,....\}$. Further $H_1=\left\{z\in C:\operatorname{Re}(z)>\frac{1}{2}\right\}$ and $H_2=\left\{z\in C:\operatorname{Re}(z)<-\frac{1}{2}\right\}$, where C is the set of all complex numbers, if $z_1\in P\cap H_1$, $z_2\in P\cap H_2$ and O represents the origin, then $\angle z_1Oz_2$ is equal to **(2013)**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$

(c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

22. Let ω be a complex cube root of unity with $\omega\neq 1$ and $P=[p_{ij}]$ be a $n\times n$ matrix with $p_{ij}=\omega^{i+j}$. Then, $P^2\neq 0$, when n is equal to **(2013)**

(a) 57 (b) 55

(c) 58 (d) 56

23. Let $a,b\in R$ and $a^2+b^2\neq 0$.

Suppose $S=\left\{z\in C: z=\frac{1}{a+ibt}, t\in R, t\neq 0\right\}$, where $i=\sqrt{-1}$. If $z=x+iy$ and $z\in S$, then (x, y) lies on **(2016)**

(a) the circle with radius $\frac{1}{2a}$ and centre $\left(\frac{1}{2a},0\right)$ for $a>0, b\neq 0$

(b) the circle with radius $-\frac{1}{2a}$ and centre $\left(-\frac{1}{2a},0\right)$ for $a<0, b\neq 0$

(c) the x-axis for $a\neq 0, b=0$

(d) the y-axis for $a=0, b\neq 0$

24. Let a, b, x and y be real numbers such that $a-b=1$ and $y\neq 0$. If the complex number $z=x+iy$ satisfies $\operatorname{Im}\left(\frac{az+b}{z+1}\right)=y$, then which of the following is(are) possible value(s) of x ? **(2017)**

(a) $-1+\sqrt{1-y^2}$ (b) $1-\sqrt{1+y^2}$

(c) $1+\sqrt{1+y^2}$ (d) $-1-\sqrt{1-y^2}$

25. For a non-zero complex number z, let arg(z) denote the principal argument with $-\pi < \arg(z) \le \pi$ Then, which of the following statement(s) is (are) FALSE? **(2018)**

(a) $\arg(-1-i) = \frac{\pi}{4}$, where $i = \sqrt{-1}$

(b) The function $f : \mathbb{R} \longrightarrow (-\pi, \pi]$ defined by $f(t) = \arg(-1 + it)$ for all $t \in \mathbb{R}$, is continuous at all points of $\mathbb{R}$, where $i = \sqrt{-1}$

(c) For any two non-zero complex number z_1 and z_2, $\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$ is an integer multiple of 2π

(d) For any three given distinct complex numbers z_1, z_2 and z_3, the locus of the point z satisfying the condition $\arg\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right) = \pi$, lies on a straight line .

26. Let s, t, r be non-zero complex numbers and L be the set of solutions $z = x + iy\left(x, y \in \mathbb{R}, i = \sqrt{-1}\right)$ of the equation $sz + t\bar{z} + r = 0$, where $\bar{z} = x - iy$. Then, which of the following statement(s) is (are) TRUE? **(2018)**

(a) If L has exactly one element, then $|s| \ne |t|$

(b) If $|s| = |t|$, then L has infinitely many elements

(c) The number of elements in $L \cap \{z : |z - 1 + i| = 5\}$ is at most 2

(d) If L has more than one element, then L has infinitely many elements .

27. Let S be the set of all complex numbers z satisfying $|z^2 + z + 1| = 1$. Then which of the following statements is/are TRUE ? **(2020)**

(a) $\left|z + \frac{1}{2}\right| \le \frac{1}{2}$ *for all* $z \in S$

(b) $|z| \le 2$ *for all* $z \in S$

(c) $\left|z + \frac{1}{2}\right| \ge \frac{1}{2}$ *for all* $z \in S$

(d) The set S has exactly four elements

28. For any complex number $w = c + id$, let $\arg(w) \in (-\pi, \pi]$, where $i = \sqrt{-1}$. Let α and β be real numbers such that for all complex numbers $z = x + iy$ satisfying $\arg\left(\frac{z+\alpha}{z+\beta}\right) = \frac{\pi}{4}$, the ordered pair (x, y) lies on the circle $x^2 + y^2 + 5x - 3y + 4 = 0$. Then which of the following statements is (are) TRUE? **(2021)**

(a) $\alpha = -1$ (b) $\alpha\beta = 4$

(c) $\alpha\beta = -4$ (d) $\beta = 4$

Numerical Value Type Questions

29. If z is any complex number satisfying $|z - 3 - 2i| \le 2$, then the minimum value of $|2z - 6 + 5i|$ is... **(2011)**

30. For any integer k, let $\alpha_k = \cos\left(\frac{k\pi}{7}\right) + i\sin\left(\frac{k\pi}{7}\right)$, where $i = \sqrt{-1}$. The value of the expression $\frac{\sum_{k=1}^{12}|\alpha_{k+1} - \alpha_k|}{\sum_{k=1}^{3}|\alpha_{4k-1} - \alpha_{4k-2}|}$ is **(2015)**

31. Let ω be a cube root of unity. Then the minimum of the set

$$\left\{\left|a+b\omega+c\omega^2\right|^2 : a,b,c \text{ are distinct non zero integers}\right\}$$

equals __________. **(2019)**

32. For a complex number z, let Re (z) denote the real part of z. Let T be the set of all complex numbers z satisfying $z^4-|z|^4=4iz^2$, where $i=\sqrt{-1}$. Then the minimum possible value of $|z_1-z_2|^2$, where $z_1,z_2\in T$ with $\text{Re}(z_1)>0$ and $\text{Re}(z_2)<0$, is... **(2020)**

Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching.For each question, choose the option corresponding to the correct matching.

33. Match the conditions/expressions in Column I with statement in Column II.

$z\neq 0$ is a complex number

	Column I		Column II
(A)	$\text{Re}(z)=0$	**(p)**	$\text{Re}(z^2)=0$
(B)	$\arg(z)=\frac{\pi}{4}$	**(q)**	$\text{Im}(z^2)=0$
		(r)	$\text{Re}(z^2)=\text{Im}(z^2)$

Options

	A	B
(a)	p	q
(b)	q	r
(c)	q	p
(d)	p	p

34. Match the statement of Column I with these in Column II.

[**Note:** Here z takes values in the complex plane and Im (z) and Re (z) denotes respectively, the imaginary part and real part of z]

	Column I		Column II
(A)	The set of points z satisfying $\|z-i\|z\|\|=\|z+i\|z\|\|$ is contained in or equal to	**(p)**	an ellipse with eccentricity 4/5
(B)	The set of points z satisfying $\|z+4\|+\|z-4\|=10$ is contained in or equal to	**(q)**	the set of points z satisfying $\text{Im}(z)=0$
(C)	If $\|w\|=2$, then the set of points $z=w-\frac{1}{w}$ is contained in or equal to	**(r)**	the set of points z satisfying $\|\text{Im}(z)\|\leq 1$
(D)	If $\|w\|=1$, then the set of points $z=w+\frac{1}{w}$ is contained in or equal to	**(s)**	the set of points satisfying $\|\text{Re}(z)\|\leq 2$
		(t)	the set of points satisfying $\|z\|\leq 3$

(2010)

Options

	A	B	C	D
(a)	t	s	r	p
(b)	q	r	t	s
(c)	p	q	r	q
(d)	q	p	t	s

35. Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i\ \sin\left(\frac{2k\pi}{10}\right)$; $k = 1,2,....,9$.

(2014)

	List I		List II
P.	For each z_k there exists a z_j such that $z_k \cdot z_j = 1$	**1.**	True
Q.	There exists a $k \in \{1, 2, ..., 9\}$ such that $z_1 \cdot z = z_k$ has no solution z in the set of complex numbers.	**2.**	False
R.	$\frac{\|1-z_1\|\|1-z_2\|...\|1-z_9\|}{10}$ equals	**3.**	1
S.	$1-\sum_{k=1}^{9}\cos\left(\frac{2k\pi}{10}\right)$ equals	**4.**	2

	P	Q	R	S
(a)	1	2	4	3
(b)	2	1	3	4
(c)	1	2	3	4
(d)	2	1	4	3

Using the following passage, solve Q.36 to Q.38

Passage – 1

Read the following passage and answer the questions.

Let A, B, C be three sets of complex number as defined below

$A = \{z : \text{Im}(z) \geq 1\}$

$B = \{z : |z-2-i| = 3\}$

$C = \{z : \text{Re}((1-i)z) = \sqrt{2}\}$ **(2008)**

36. The number of elements in the set $A \cap B \cap C$ is

(a) 0 (b) 1

(c) 2 (d) ∞

37. Let z be any point in $A \cap B \cap C$. Then $|z+1-i|^2 + |z-5-i|^2$ lies between

(a) 25 and 29 (b) 30 and 34

(c) 35 and 39 (d) 40 and 44

38. Let z be any point in $A \cap B \cap C$ and let w be any point satisfying $|w-2-i| < 3$. Then, $|z| - |w| + 3$ lies between

(a) –6 and 3 (b) –3 and 6

(c) –6 and 6 (d) – 3 and 9

Using the following passage, solve Q.39 and Q.40

Passage – 2

Let $S = S_1 \cap S_2 \cap S_3$, where

$S_1 = \{z \in C : |z| < 4\}$, $S_2 = \left\{z \in C : \text{Im}\left[\frac{z-1+\sqrt{3}\,i}{1-\sqrt{3}\,i}\right] > 0\right\}$

and $S_3 : \{z \in C : \text{Re}(z) > 0\}$ **(2013)**

39. Area of S is equal to

(a) $\frac{10\pi}{3}$ (b) $\frac{20\pi}{3}$

(c) $\frac{16\pi}{3}$ (d) $\frac{32\pi}{3}$

40. $\min_{z \in s} |1-3i-z|$ is equal to

(a) $\frac{2-\sqrt{3}}{2}$ (b) $\frac{2+\sqrt{3}}{2}$

(c) $\frac{3-\sqrt{3}}{2}$ (d) $\frac{3+\sqrt{3}}{2}$

Answer Key

CHAPTER -2 | COMPLEX NUMBERS

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

DIRECTION TO USE -
Scan the QR code and check detailed solutions.

1. (b)	**2.** (a)	**3.** (b)	**4.** (d)	**5.** (b)
6. (b)	**7.** (d)	**8.** (d)	**9.** (d)	**10.** (b)
11. (b)	**12.** (c)	**13.** (c)	**14.** (d)	**15.** (a)
16. (c)	**17.** (b)	**18.** (c)	**19.** (c)	**20.** (a)
21. (b)	**22.** (d)	**23.** (c)	**24.** (d)	**25.** (d)
26. (b)	**27.** (a)	**28.** (c)	**29.** (c)	**30.** (d)
31. (b)	**32.** (a)	**33.** (c)	**34.** (a)	**35.** (b)
36. (c)	**37.** (b)	**38.** (b)	**39.** (c)	**40.** (d)
41. (a)	**42.** (c)	**43.** (a)	**44.** (a)	**45.** (b)
46. (a)	**47.** (b)	**48.** (b)	**49.** (b)	**50.** (b)
51. (b)	**52.** (b)	**53.** (4)	**54.** (-2)	**55.** (0)
56. (0)	**57.** (-4)	**58.** (-2)	**59.** (3)	**60.** (1)
61. (3)	**62.** (6)	**63.** (1)	**64.** (1.5)	**65.** (17)

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

DIRECTION TO USE -
Scan the QR code and check detailed solutions.

1. (c)	**2.** (d)	**3.** (a)	**4.** (c)	**5.** (d)
6. (b)	**7.** (a)	**8.** (b)	**9.** (c)	**10.** (d)
11. (c)	**12.** (c)	**13.** (a)	**14.** (c)	**15.** (c)
16. (d)	**17.** (a)	**18.** (a)	**19.** (91)	**20.** (b)
21. (a)	**22.** (a)	**23.** (c)	**24.** (b)	**25.** (4.00)
26. (a)	**27.** (c)	**28.** (c)	**29.** (d)	**30.** (c)
31. (a)	**32.** (a)	**33.** (c)	**34.** (a)	**35.** (c)
36. (c)	**37.** (b)	**38.** (a)	**39.** (48.00)	
40. (3.00)	**41.** (b)	**42.** (c)	**43.** (310.00)	
44. (10.00)	**45.** (d)	**46.** (6.00)	**47.** (b)	**48.** (0.00)
49. (a)	**50.** (4.00)	**51.** (b)	**52.** (c)	**53.** (c)
54. (c)	**55.** (a)	**56.** (d)	**57.** (1.00)	**58.** (c)
59. (1.00)	**60.** (5.00)	**61.** (6.00)	**62.** (c)	**63.** (c)
64. (13.00)	**65.** (6.00)	**66.** (98.00)		**67.** (b)
68. (c)	**69.** (d)	**70.** (a)		

CHAPTER -2 | COMPLEX NUMBERS

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

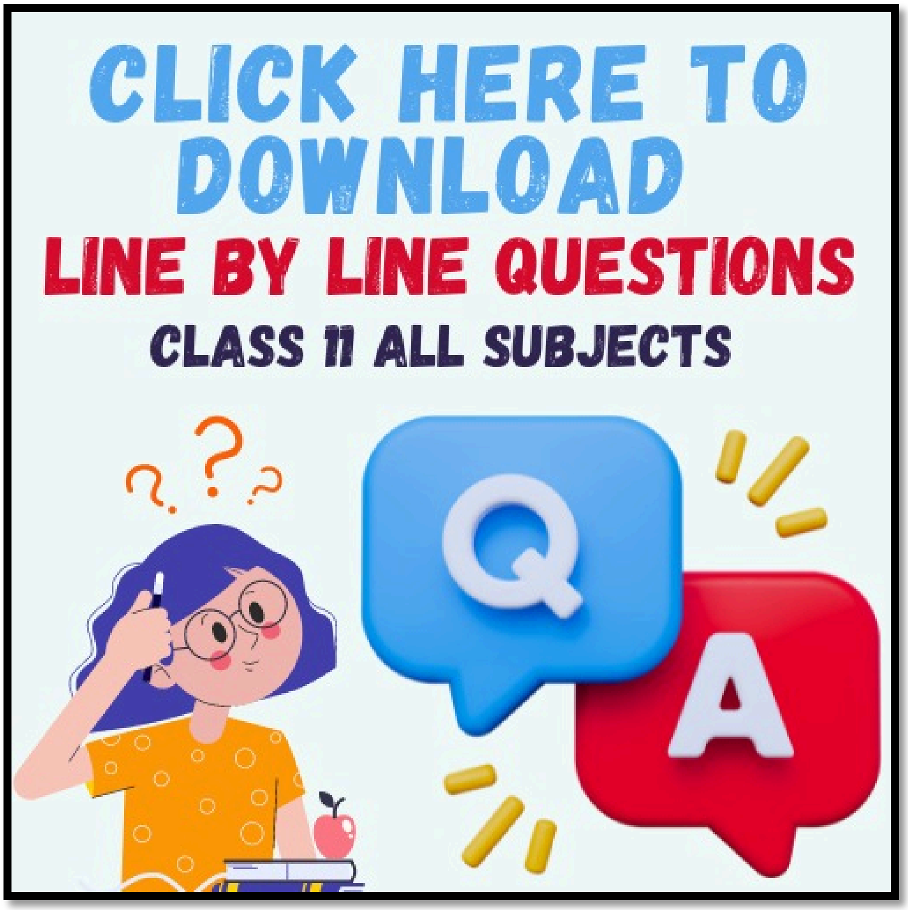
DIRECTION TO USE - Scan the QR code and check detailed solutions.

1. (c) **2.** (b) **3.** (c) **4.** (a) **5.** (b)
6. (c) **7.** (a) **8.** (c) **9.** (c) **10.** (b)
11. (b) **12.** (b) **13.** (c) **14.** (a) **15.** (b)
16. (d) **17.** (a) **18.** (a) **19.** (b) **20.** (a)
21. (a) **22.** (b) **23.** (a) **24.** (d) **25.** (c)
26. (d) **27.** (c) **28.** (c) **29.** (b) **30.** (d)
31. (a) **32.** (d) **33.** (d) **34.** (a,b,c)
35. (a,d) **36.** (a,b,c) **37.** (a,b,c)
38. (a,c,d) **39.** (a,b) **40.** (a,c) **41.** (b,c)
42. (a,b,c,d) **43.** (a,b) **44.** (2) **45.** (12)
46. (2) **47.** (1) **48.** (2) **49.** (1) **50.** (4)
51. (1) **52.** (4) **53.** (1) **54.** (8) **55.** (a)
56. (b) **57.** (a) **58.** (d) **59.** (d) **60.** (b)
61. (a) **62.** (b) **63.** (d) **64.** (b) **65.** (c)

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

DIRECTION TO USE - Scan the QR code and check detailed solutions.

1. (d) **2.** (c) **3.** (b) **4.** (b) **5.** (a)
6. (b) **7.** (c) **8.** (a) **9.** (b) **10.** (d)
11. (d) **12.** (d) **13.** (d) **14.** (a) **15.** (a)
16. (d) **17.** (c) **18.** (c) **19.** (c)
20. (a,c,d) **21.** (c,d) **22.** (b,c,d) **23.** (a,c,d) **24.** (a,d)
25. (a,b,d) **26.** (a,c,d) **27.** (b,c) **28.** (b,d) **29.** (5)
30. (4) **31.** (3.00) **32.** (8.00) **33.** (c) **34.** (d)
35. (c) **36.** (b) **37.** (c) **38.** (d) **39.** (b)
40. (c)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We are thrilled to introduce the School of Educators WhatsApp Group, a platform designed exclusively for educators to enhance your teaching & Learning experience and learning outcomes. Here are some of the key benefits you can expect from joining our group:

BENEFITS OF SOE WHATSAPP GROUPS

- **Abundance of Content:** Members gain access to an extensive repository of educational materials tailored to their class level. This includes various formats such as PDFs, Word files, PowerPoint presentations, lesson plans, worksheets, practical tips, viva questions, reference books, smart content, curriculum details, syllabus, marking schemes, exam patterns, and blueprints. This rich assortment of resources enhances teaching and learning experiences.

- **Immediate Doubt Resolution:** The group facilitates quick clarification of doubts. Members can seek assistance by sending messages, and experts promptly respond to queries. This real-time interaction fosters a supportive learning environment where educators and students can exchange knowledge and address concerns effectively.

- **Access to Previous Years' Question Papers and Topper Answers:** The group provides access to previous years' question papers (PYQ) and exemplary answer scripts of toppers. This resource is invaluable for exam preparation, allowing individuals to familiarize themselves with the exam format, gain insights into scoring techniques, and enhance their performance in assessments.

- **Free and Unlimited Resources:** Members enjoy the benefit of accessing an array of educational resources without any cost restrictions. Whether its study materials, teaching aids, or assessment tools, the group offers an abundance of resources tailored to individual needs. This accessibility ensures that educators and students have ample support in their academic endeavors without financial constraints.

- **Instant Access to Educational Content:** SOE WhatsApp groups are a platform where teachers can access a wide range of educational content instantly. This includes study materials, notes, sample papers, reference materials, and relevant links shared by group members and moderators.

- **Timely Updates and Reminders:** SOE WhatsApp groups serve as a source of timely updates and reminders about important dates, exam schedules, syllabus changes, and academic events. Teachers can stay informed and well-prepared for upcoming assessments and activities.

- **Interactive Learning Environment:** Teachers can engage in discussions, ask questions, and seek clarifications within the group, creating an interactive learning environment. This fosters collaboration, peer learning, and knowledge sharing among group members, enhancing understanding and retention of concepts.

- **Access to Expert Guidance:** SOE WhatsApp groups are moderated by subject matter experts, teachers, or experienced educators can benefit from their guidance, expertise, and insights on various academic topics, exam strategies, and study techniques.

Join the School of Educators WhatsApp Group today and unlock a world of resources, support, and collaboration to take your teaching to new heights. To join, simply click on the group links provided below or send a message to +91-95208-77777 expressing your interest.

Together, let's empower ourselves & Our Students and inspire the next generation of learners.

Best Regards,
Team
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Groups Rules & Regulations:

To maximize the benefits of these WhatsApp groups, follow these guidelines:

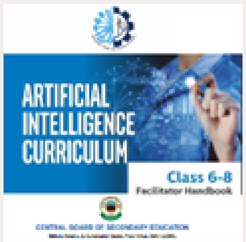
1. Share your valuable resources with the group.
2. Help your fellow educators by answering their queries.
3. Watch and engage with shared videos in the group.
4. Distribute WhatsApp group resources among your students.
5. Encourage your colleagues to join these groups.

Additional notes:

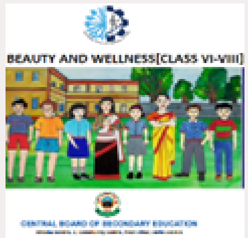
1. Avoid posting messages between 9 PM and 7 AM.
2. After sharing resources with students, consider deleting outdated data if necessary.
3. It's a NO Nuisance groups, single nuisance and you will be removed.
   - No introductions.
   - No greetings or wish messages.
   - No personal chats or messages.
   - No spam. Or voice calls
   - Share and seek learning resources only.

Please only share and request learning resources. For assistance, contact the helpline via WhatsApp: +91-95208-77777.

SKILL MODULES BEING OFFERED IN MIDDLE SCHOOL

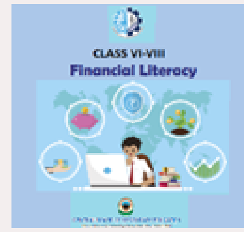


Artificial Intelligence



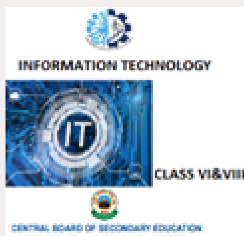
Beauty & Wellness

Design Thinking & Innovation



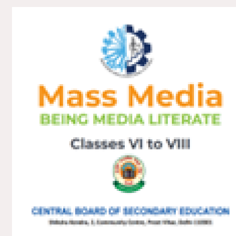
Financial Literacy

Handicrafts

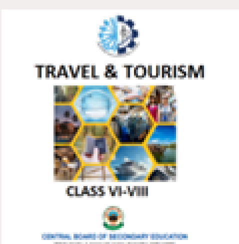


Information Technology

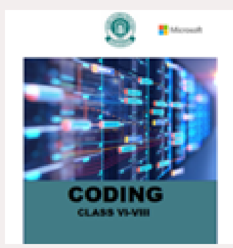
Marketing/Commercial Application



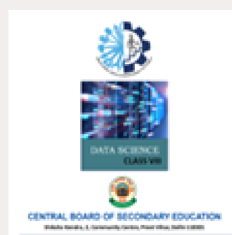
Mass Media - Being Media Literate



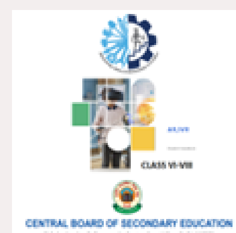
Travel & Tourism



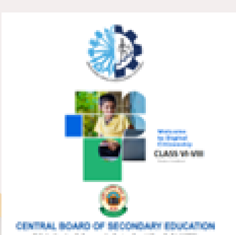
Coding



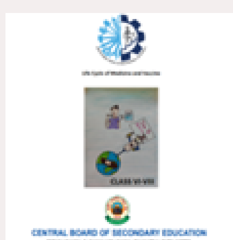
Data Science (Class VIII only)



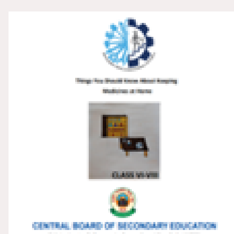
Augmented Reality / Virtual Reality



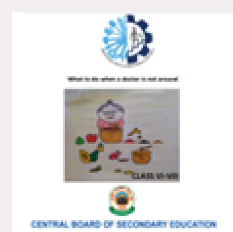
Digital Citizenship



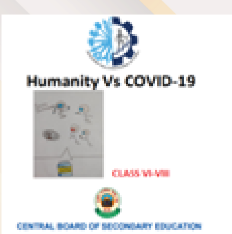
Life Cycle of Medicine & Vaccine



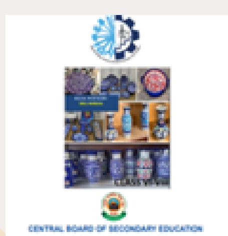
Things you should know about keeping Medicines at home



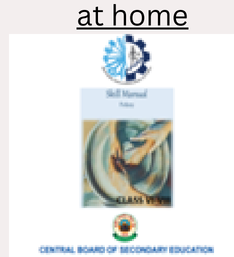
What to do when Doctor is not around



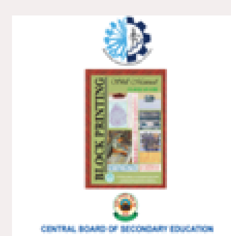
Humanity & Covid-19



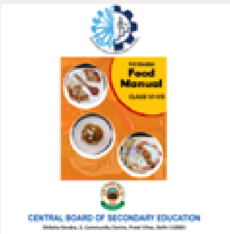
Blue Pottery



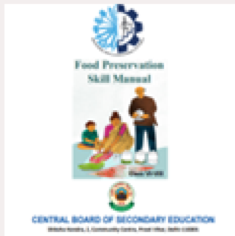
Pottery



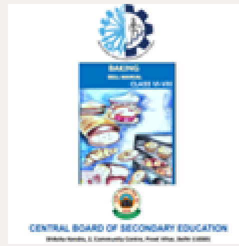
Block Printing



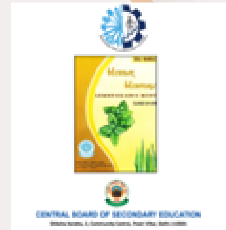
[Food](#)



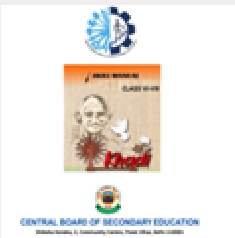
[Food Preservation](#)



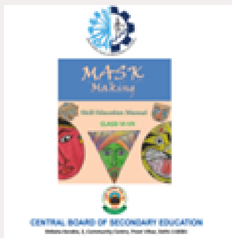
[Baking](#)



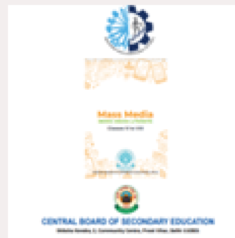
[Herbal Heritage](#)



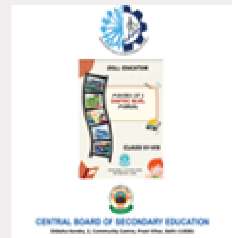
[Khadi](#)



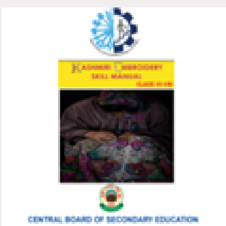
[Mask Making](#)



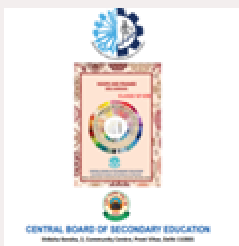
[Mass Media](#)



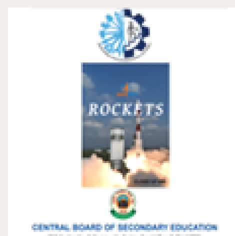
[Making of a Graphic Novel](#)



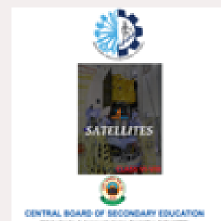
[Kashmiri Embroidery](#)



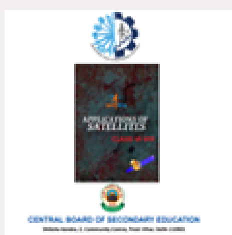
[Embroidery](#)



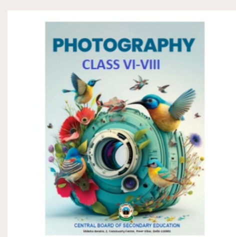
[Rockets](#)



[Satellites](#)

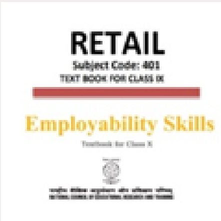


[Application of Satellites](#)



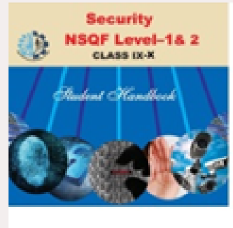
[Photography](#)

SKILL SUBJECTS AT SECONDARY LEVEL (CLASSES IX – X)

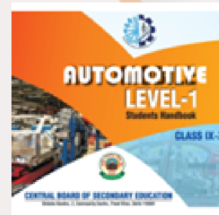


Retail

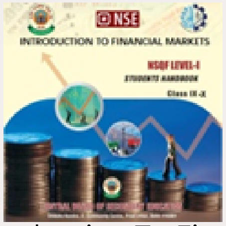
Information Technology



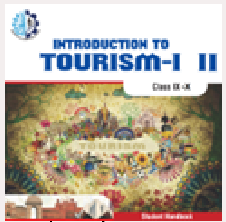
Security



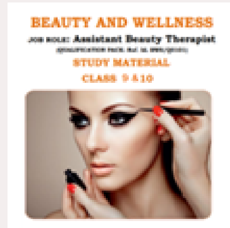
Automotive



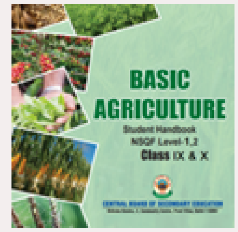
Introduction To Financial Markets



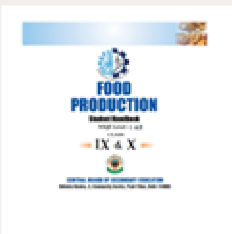
Introduction To Tourism



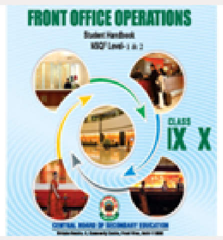
Beauty & Wellness



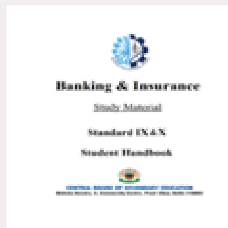
Agriculture



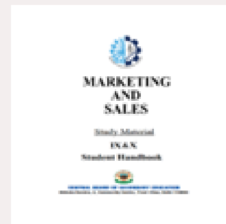
Food Production



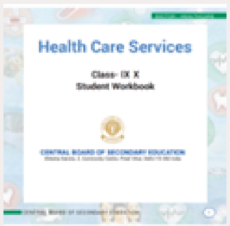
Front Office Operations



Banking & Insurance

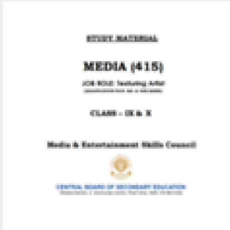


Marketing & Sales

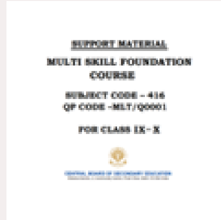


Health Care

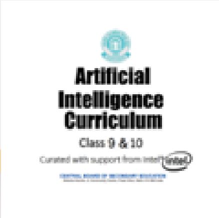
Apparel



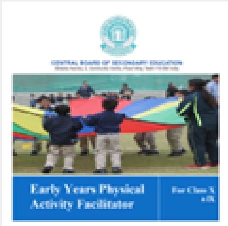
Multi Media



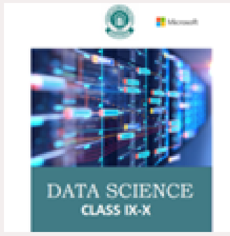
Multi Skill Foundation Course



Artificial Intelligence

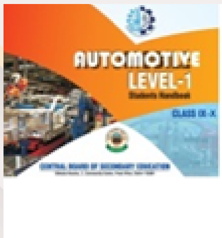


Physical Activity Trainer

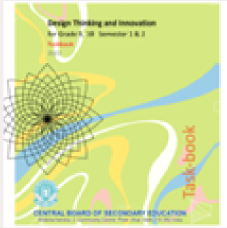


Data Science

Electronics & Hardware (NEW)

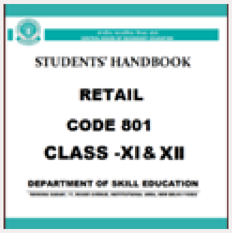


Foundation Skills For Sciences (Pharmaceutical & Biotechnology)(NEW)

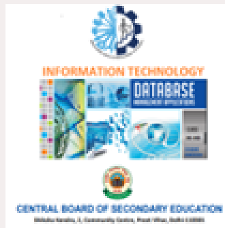


Design Thinking & Innovation (NEW)

SKILL SUBJECTS AT SR. SEC. LEVEL (CLASSES XI – XII)

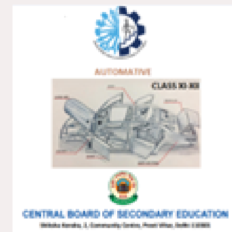


Retail



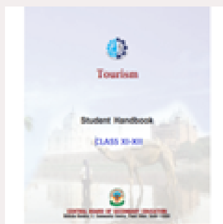
InformationTechnology

Web Application

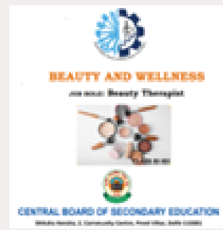


Automotive

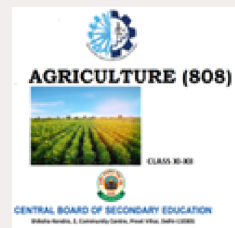
Financial Markets Management



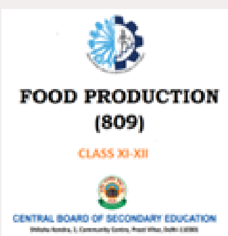
Tourism



Beauty & Wellness

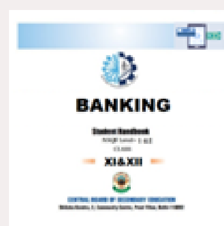


Agriculture

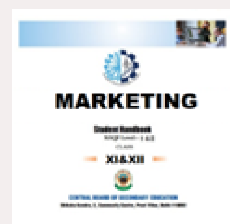


Food Production

Front Office Operations

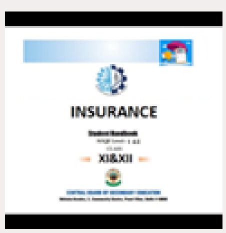


Banking

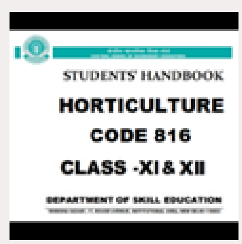


Marketing

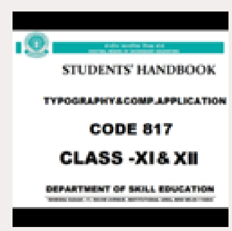
Health Care



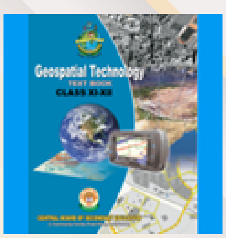
Insurance



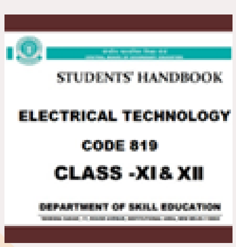
Horticulture



Typography & Comp. Application

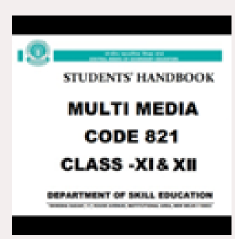


Geospatial Technology

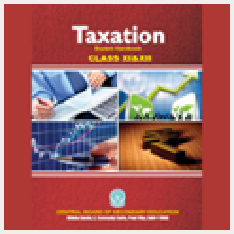


Electrical Technology

Electronic Technology

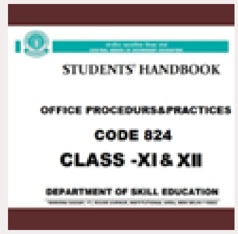


Multi-Media

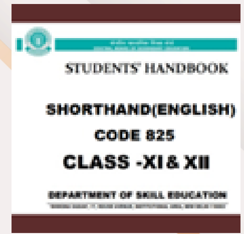


Taxation

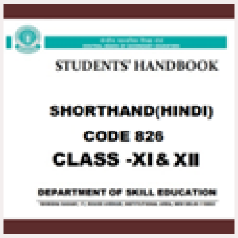
Cost Accounting



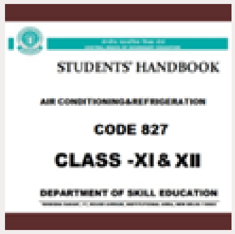
Office Procedures & Practices



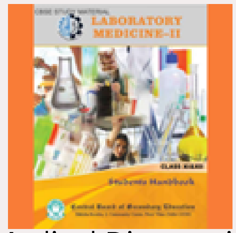
Shorthand (English)



Shorthand (Hindi)

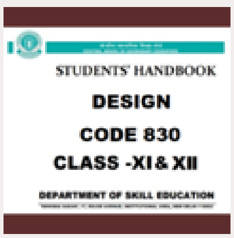


Air-Conditioning & Refrigeration

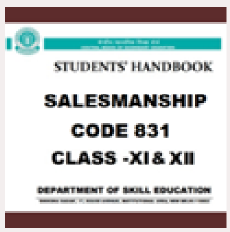


Medical Diagnostics

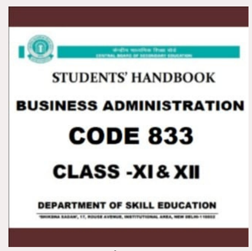
Textile Design



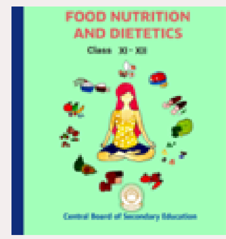
Design



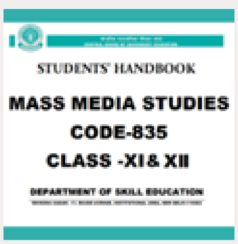
Salesmanship



Business Administration



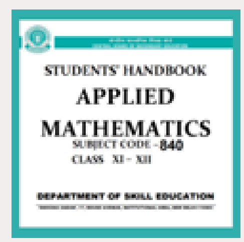
Food Nutrition & Dietetics



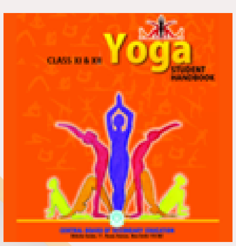
Mass Media Studies

Library & Information Science

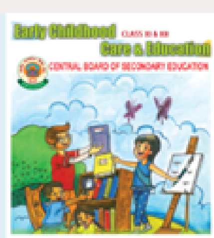
Fashion Studies



Applied Mathematics



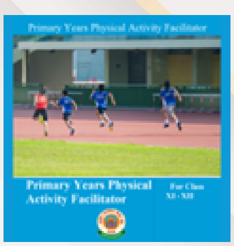
Yoga



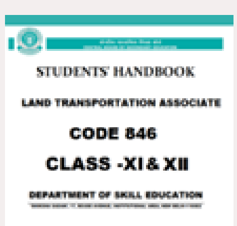
Early Childhood Care & Education

Artificial Intelligence

Data Science



Physical Activity Trainer(new)



Land Transportation Associate (NEW)

Electronics & Hardware (NEW)

Design Thinking & Innovation (NEW)